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Research article

Interaction of mechanical, temperature and diffusion fields in a solid cylinder subjected to transient heating

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The article examines the phenomena caused by the mutual influence of mechanical, temperature and diffusion fields in a continuous orthotropic multicomponent cylinder under the action of unsteady external perturbations set on the cylinder surface. The cylinder does not experience volumetric perturbations. It is assumed that the physico-mechanical processes occurring in the cylinder depend only on the radial coordinate. The mathematical formulation involves a system of coupled nonstationary thermodiffusion equations in a cylindrical coordinate system. The employed heat and mass transfer model accounts for the finite velocity of thermal and diffusion wave propagation. The kinematic relations for thermal and diffusion flows are constructed based on the generalized Lord–Shulman theory. The proposed algorithm for solving a polar-symmetric problem is based on the integral representation of the desired fields in the form of time-dependent convolutions of Green's functions and the functions, describing surface thermomechanodiffusion perturbations. The Green's functions are found by applying the time-domain Laplace transform and the Fourier–Bessel series expansion, which makes it possible to reduce the initial boundary value problem to a system of linear algebraic equations with respect to the coefficients of the Fourier series. The solutions of this system are rational functions, and therefore the inverse Laplace transformation is carried out analytically using the theory of residues and tables of operational calculus. Using a three-component cylinder as an example, a numerical analysis of the interaction between mechanical, temperature and diffusion fields under external unsteady thermal action is performed and the obtained results are compared with the solutions to known problems of thermoelasticity and mechanodiffusion. The effect of the finite velocity of propagation of thermal and diffusion fluxes on the kinetics of heat and mass transfer in continuous media under different types of external thermal load is investigated.

Keywords: thermomechanodiffusion, phase-lag effects, relaxation of heat and diffusion flows, unsteady problems, Green's functions, Laplace transform, Fourier series, Bessel functions

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1. Introduction

The present study is devoted to modeling the nonstationary interaction of mechanical, thermal, and diffusion fields in cylindrical bodies. The investigation of processes arising from the coupling of these fields is important not only from a theoretical standpoint but also from a practical perspective, since cylinders constitute the basis of various engineering systems such as pipelines, oil and gas transmission systems, heating systems, shafts, bushings, and others.

The question of mutual influence between fields of different physical nature has a long history. Effects caused by the interaction of mechanical and thermal fields were discussed as early as the late 19th century [1], whereas a consistent theory of coupled thermoelasticity, including a hyperbolic-type equation of motion and a parabolic heat conduction equation, was developed only in the mid-20th century [2]. The first studies on the coupling of mechanical and diffusion fields appeared in the first half of the 20th century [3, 4] and were predominantly experimental in nature. In the second half of the 20th century, mathematical models of heat and mass transfer in continua were developed [5–7], in which the diffusion equation was also parabolic. At the same time, studies conducted in the second half of the 20th century revealed a significant drawback of the classical thermomechanodiffusion model described above, namely, it predicted different propagation speeds: finite for elastic waves and infinite for thermal and diffusion pulses. As a result, various

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modifications of the classical theory of thermoelastic diffusion were proposed, based on the wave nature of heat and mass propagation, implying finite propagation velocities for thermal and diffusion fluxes.

A fairly comprehensive review of publications devoted to modern theories of heat and mass transfer is presented in [8, 9], where considerable interest is noted in problems related to accounting for finite propagation speeds of thermal and diffusion disturbances. These papers provide references to experimental studies confirming the existence (albeit at very small time scales) of wave processes of heat and mass transfer in metals, crystals, polymers, biological tissues, and mixtures. Data on relaxation times for a number of materials are presented, and the applicability limits of various heat and mass transfer models are discussed in detail.

Among the first models employing hyperbolic-type equations to describe heat transfer are those of Cattaneo, Lord–Shulman, Green–Lindsay, and others [10–14]. Subsequently, these models were extended to describe mass transfer in continua, and at present almost all models of heat and mass transfer account for the finite propagation speed of thermal and diffusion disturbances [15–32]. The works cited here focus on problems of thermomechanodiffusion in cylindrical and spherical coordinate systems. Such publications are significantly fewer than those devoted to problems in the rectangular (Cartesian) coordinate system.

When solving nonstationary problems for continua in curvilinear coordinates, integral transforms are typically employed: the Laplace transform in time and the Fourier and Hankel transforms in space, as demonstrated in [15, 18, 19, 21–25]. Given the complexity associated with inverting the Laplace transform in coupled mechanics problems, approximate numerical-analytical methods are used to obtain the original functions, such as the Durbin method (and its modifications) [18, 19, 22–24], the Gaver–Stehfast algorithm [25], and quadrature formulas based on Riemann sums [15].

Without discussing the advantages and disadvantages of these approaches, it should be noted that Laplace-domain expressions arising in the solution of coupled thermomechanodiffusion problems are so cumbersome that verifying the correctness of a particular inversion method is often impractical. The issue of the validity of approximate Laplace transform inversion methods is discussed in detail in [33].

On the other hand, for certain boundary conditions, the method of separation of variables can be applied to solve coupled thermomechanodiffusion problems, reducing the inversion of the Laplace transform of the original problem to that of a rational function. In this case, the original functions can be determined analytically using residues and tables of operational calculus [34–36], which, in general, does not require additional verification.

In this paper, a model is proposed that describes one-dimensional nonstationary thermomechanodiffusion processes in a solid multicomponent cylinder. This model generalizes previously considered mechanodiffusion models [34, 35] and enables the analysis of the interaction of mechanical, thermal, and diffusion processes in continua.

2. General problem formulation

We consider the problem of nonstationary thermomechanodiffusion in a multicomponent solid cylinder of radius R_0, subjected to external loading applied to its surface. Surface loads induce deformation heating and the accompanying diffusion, i.e., redistribution of component concentrations under the action of thermoelastic deformations. A linear model of coupled thermoelastic diffusion is used to describe the physical and mechanical processes in the continuum, represented by the following system of equations [37–39]:

$$\rho\frac{\partial^2 u^i}{\partial t^2}=\nabla_j\sigma^{ij},\quad \rho T_0\frac{\partial S}{\partial t}=-\nabla_i q^i,\quad \frac{\partial \eta^{(q)}}{\partial t}=-\nabla_i J^{i(q)},$$
$$\eta^{(N+1)}=-\sum_{q=1}^{N}\eta^{(q)}\quad \big(i,j=\overline{1,3},\ q=\overline{1,N}\big). \tag{1}$$

Here t denotes time; T_0 is the initial temperature of the continuum; ∇_i is the covariant differentiation operator; u^i are the components of the mechanical displacement vector; σ^{ij} are the components of the stress tensor; $n_0^{(q)}$ and $n^{(q)}$ are the initial and current concentrations (mass fractions) of the q-th component in an $(N+1)$-component continuum; $\eta^{(q)}=n^{(q)}-n_0^{(q)}$ is the concentration increment; ρ is the density; q^i are the components of the heat flux vector; $J^{i(q)}$ are the components of the diffusion flux vector of the q-th component; S is the specific entropy.

The system (1) is supplemented by zero initial conditions corresponding to the undisturbed state of the cylinder at the initial time $t=0$, and by boundary conditions for displacements, thermal, and diffusion fluxes on the cylinder surface Π:

$$u^i\Big|_{\Pi}=U^i, \qquad q^i\nu_i\Big|_{\Pi}=Q_0, \qquad J^{i(q)}\nu_i\Big|_{\Pi}=I^{(q)},$$

where U^i, Q_0, $I^{(q)}$ are prescribed surface perturbations, and ν^i are the components of the normal vector to the surface.

The model employed accounts for the finite propagation speed of thermal and diffusion fluxes and generalizes the Lord–Shulman theory of thermoelasticity, which in turn is based on the modified Fourier law in the Vernotte–Cattaneo–Lykov form. For thermomechanodiffusion processes, the constitutive relations for entropy and stress tensor components, as well as the Fourier and Fick laws, are written as follows [19, 21, 23, 25, 26, 37]:

$$\begin{gathered}
\sigma^{ij}=C^{ijkl}\nabla_k u_l-b^{ij}\vartheta-\sum_{p=1}^{N}\alpha^{(p)ij}\eta^{(p)},\\
S=\frac{c_0}{T_0}\vartheta+\frac{b^{ij}}{\rho}\nabla_i u_j+R\sum_{j=1}^{N}\frac{\ln(n_0^{(j)}\gamma^{(j)})}{m^{(j)}}\eta^{(j)}, \qquad \sum_{k=0}^{K}\frac{(\tau_\vartheta)^k}{k!}\frac{\partial^k q^i}{\partial t^k}=-\kappa^{ij}\nabla_j\vartheta,\\
\sum_{k=0}^{M}\frac{(\tau_\eta^{(q)})^k}{k!}\frac{\partial^{k+1}J^{i(q)}}{\partial t^{k+1}}=\frac{m^{(q)}n_0^{(q)}D^{(q)ij}}{\rho RT_0}\nabla_j\left(\alpha^{(q)kl}\nabla_k u_l\right)+\\
+\frac{n_0^{(q)}D^{(q)ij}}{T_0}\ln\left(n_0^{(q)}\gamma^{(q)}\right)\nabla_j\vartheta-\sum_{p=1}^{N}D^{(p)ij}g^{(qp)}\nabla_j\eta^{(r)}.
\end{gathered} \tag{2}$$

Here $\vartheta=T-T_0$ is the temperature increment; T is the current temperature of the continuum; τ_ϑ is the relaxation time of heat fluxes; $\tau_\eta^{(q)}$ is the relaxation time of diffusion fluxes; $D^{(q)ij}$ are the diffusion coefficients; κ^{ij} are the components of the thermal conductivity tensor; $m^{(q)}$ is the molar mass of the q-th component; $g^{(qr)}$ is the Darken thermodynamic factor; $\gamma^{(q)}$ is the activity coefficient; c_0 is the heat capacity coefficient in the initial state; C^{ijkl}, b^{ij}, and $\alpha^{(r)ij}$ are the components of the tensors of elastic, thermal, and diffusion constants; R is the universal gas constant.

For $K=0$, $M=0$, the classical Fourier and Fick laws are recovered with coupling between mechanical, thermal, and diffusion fields. The case $K=1$, $M=1$ corresponds to the Vernotte–Cattaneo–Lykov approximation. More complex modifications of Fourier and Fick laws also exist, namely theories of heat and mass transfer with two or more relaxation times as parameters (such as the Green–Naghdi and Moore–Gibson–Thompson theories).

Without attempting to assess the merits or drawbacks of various approaches to modeling wave processes of heat and mass transfer in continua, we note only that models with multiple relaxation times remain insufficiently studied due to the lack of reliable data on their parameters. Even for the model with one relaxation time for each flux (2) used in this work, only approximate estimates of relaxation times are available. For example, in [14] the relaxation time of the heat flux τ_ϑ is considered in the range $10^{-14}\div10^{-7}$ s, in [40] — $\tau_\vartheta=10^{-12}\div10^{-10}$ s, and in [41] — $\tau_\vartheta=10^{-12}\div10^{-6}$ s. It is also stated in [41], with reference to [42], that the relaxation time of diffusion fluxes $\tau_\eta^{(q)}$ in polymers «amounts to several seconds». Experimental data in [43] indicate $\tau_\vartheta=20$ s for sand and $\tau_\vartheta=28.7$ s for soda. At the same time, in [44] the values for sand are given as $\tau_\vartheta=2.26$ s and for soda as $\tau_\vartheta=0.66$ s (a difference of two orders of magnitude compared to the previous estimate).

In the present study, we adopt intermediate values of relaxation times that are most commonly used in modeling coupled thermomechanodiffusion processes with finite propagation speeds of heat and mass transfer. Following [19, 20, 22, 23, 26], the relaxation time of heat flux is taken to be on the order of hundredths of a second, while the relaxation times of diffusion fluxes are on the order of tenths of a second.

3. One-dimensional model in cylindrical coordinates

We assume that the cylinder material is orthotropic (Voigt notation is used for physical constants) and represents an ideal solid solution in which cross-diffusion effects caused by mutual influence of diffusion fluxes of different components are neglected. In this case, in (2) the activity coefficient and the Darken thermodynamic factor take the following forms:

$\gamma^{(q)}=1$, $g^{(qr)}=\delta_{qr}$, where δ_{qr} is the Kronecker delta. Moreover, all physical fields depend only on a single spatial coordinate. Then $\mathbf{u}=\{u_r(r,t),0,0\}$, $\vartheta=\vartheta(r,t)$, $\eta^{(q)}=\eta^{(q)}(r,t)$.

Taking into account these assumptions and relations (2), the system (1) in cylindrical coordinates takes the following form (hereinafter the equation for $\eta^{(N+1)}$ is omitted due to its cumbersome form):

$$\rho\frac{\partial^2 u_r}{\partial t^2}=C_{11}\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right)-b_1\frac{\partial\vartheta}{\partial r}-\sum_{j=1}^{N}\alpha_1^{(j)}\frac{\partial\eta^{(j)}}{\partial r},$$

$$\left(\sum_{k=0}^{K}\frac{\tau_\vartheta^k}{k!}\frac{\partial^{k+1}}{\partial t^{k+1}}\right)\left[\rho c_0\vartheta+T_0 b_1\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)+\rho R T_0\sum_{j=1}^{N}\frac{\ln(n_0^{(j)}\gamma^{(j)})}{m^{(i)}}\eta_j\right]=\kappa_1\left(\frac{\partial^2\vartheta}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta}{\partial r}\right),$$

$$\sum_{k=0}^{M}\frac{(\tau_\eta^{(q)})^k}{k!}\frac{\partial^{k+1}}{\partial t^{k+1}}\eta^{(q)}=-\Lambda_{11}^{(q)}\left(\frac{\partial^3 u_r}{\partial r^3}+\frac{2}{r}\frac{\partial^2 u_r}{\partial r^2}-\frac{1}{r^2}\frac{\partial u_r}{\partial r}+\frac{u_r}{r^3}\right)+D_1^{(q)}\left(\frac{\partial^2\eta^{(q)}}{\partial r^2}+\frac{1}{r}\frac{\partial\eta^{(q)}}{\partial r}\right)-M_1^{(q)}\left(\frac{\partial^2\vartheta}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta}{\partial r}\right),$$

where

$$C_{11}=C_{1111},\quad b_1=b_{11},\quad D_1^{(q)}=D_{11}^{(q)},\quad \alpha_1^{(q)}=\alpha_{11}^{(q)},\quad \Lambda_{11}^{(q)}=\frac{m^{(q)}n_0^{(q)}D_1^{(q)}\alpha_1^{(q)}}{\rho R T_0},\quad M_1^{(q)}=\frac{n_0^{(q)}D_1^{(q)}}{T_0}\ln(n_0^{(q)}\gamma^{(q)}).$$

The initial and boundary conditions are written as follows:

$$u_r\Big|_{t=0}=0,\qquad \frac{\partial u_r}{\partial t}\Big|_{t=0}=0,\qquad \frac{\partial^k\vartheta}{\partial t^k}\Big|_{t=0}=0,\qquad \frac{\partial^m\eta^{(q)}}{\partial t^m}\Big|_{t=0}=0\qquad \left(k=\overline{0,K},\ m=\overline{0,M}\right),$$

$$u_r\Big|_{r=R_0}=f_1(t),\qquad \frac{\partial\vartheta}{\partial r}\Big|_{r=R_0}=f_2(t),\qquad \left[\Lambda_{11}^{(q)}\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right)+M_1^{(q)}\frac{\partial\vartheta}{\partial r}-D_1^{(q)}\frac{\partial\eta^{(q)}}{\partial r}\right]\Big|_{r=R_0}=f_{q+2}(t).$$

Let us introduce dimensionless variables (denoted by the symbol "*", which is omitted hereafter):

$$r^*=\frac{r}{R_0},\quad \tau=\frac{ct}{R_0},\quad u=\frac{u_r}{R_0},\quad \vartheta^*=\frac{\vartheta}{T_0},\quad \eta_q=\eta^{(q)},\quad c^2=\frac{C_{11}}{\rho},\quad b=\frac{T_0}{\rho c^2}b_1,\quad \alpha_q=\frac{\alpha_1^{(q)}}{C_{11}},\quad \kappa=\frac{\kappa_1}{\rho c_0 R_0 c},$$

$$\tau_0=\frac{c}{R_0}\tau_\vartheta,\quad \tau_q=\frac{c}{R_0}\tau_\eta^{(q)},\quad \Lambda_q=\frac{\Lambda_{11}^{(q)}}{cR_0},\quad \beta_i=\frac{R}{c_0}\frac{\ln(n_0^{(i)}\gamma^{(i)})}{m^{(i)}},\quad B=\frac{b_1}{\rho c_0},\quad D_q=\frac{D_1^{(q)}}{cR_0},\quad M_q=\frac{T_0}{cR_0}M_1^{(q)},$$

$$C_{ij}^*=\frac{C_{ij}}{C_{11}},\quad f_1^*(\tau)=\frac{f_1(R_0\tau/c)}{R_0},\quad f_2^*(\tau)=\frac{f_2(R_0\tau/c)}{T_0},\quad f_{q+2}^*(\tau)=\frac{f_{q+2}(R_0\tau/c)}{c}.$$

As a result, the mathematical formulation of the problem becomes:

$$\frac{\partial^2 u}{\partial\tau^2}=\left(u''+\frac{u'}{r}-\frac{u}{r^2}\right)-b\vartheta'-\sum_{j=1}^{N}\alpha_j\eta_j',$$

$$\left(\sum_{k=0}^{K}\frac{\tau_0^k}{k!}\frac{\partial^{k+1}}{\partial\tau^{k+1}}\right)\left[\vartheta+B\left(u'+\frac{u}{r}\right)+\sum_{j=1}^{N}\beta_j\eta_j\right]=\kappa\left(\vartheta''+\frac{\vartheta'}{r}\right),\tag{3}$$

$$\sum_{k=0}^{M}\frac{\tau_q^k}{k!}\frac{\partial^{k+1}\eta_q}{\partial\tau^{k+1}}=-\Lambda_q\left(u'''+\frac{2u''}{r}-\frac{u'}{r^2}+\frac{u}{r^3}\right)+D_q\left(\eta_q''+\frac{\eta_q'}{r}\right)-M_q\left(\vartheta''+\frac{\vartheta'}{r}\right)\qquad \left(q=\overline{1,N}\right),$$

$$u\Big|_{\tau=0}=0,\quad \frac{\partial u}{\partial\tau}\Big|_{\tau=0}=0,\quad \frac{\partial^k\vartheta}{\partial\tau^k}\Big|_{\tau=0}=0,\quad \frac{\partial^m\eta_q}{\partial\tau^m}\Big|_{\tau=0}=0\qquad \left(k=\overline{0,K},\quad m=\overline{0,M}\right),$$

$$u\Big|_{r=1}=f_1(\tau),\quad \vartheta'\Big|_{r=1}=f_2(\tau),\quad \left[\Lambda_q\left(u''+\frac{u'}{r}-\frac{u}{r^2}\right)+M_q\vartheta'-D_q\eta_q'\right]\Big|_{r=1}=f_{q+2}(\tau),\tag{4}$$

where the prime denotes differentiation with respect to the spatial coordinate r.

4. Solution algorithm

The solution of the initial-boundary value problem (3), (4) is represented in the form of time convolutions:

$$\left\{\begin{array}{c} u(r,\tau) \\ \vartheta(r,\tau) \\ \eta_q(r,\tau) \end{array}\right\} = \sum_{m=1}^{N+2} \int_0^{\tau} \left\{\begin{array}{c} G_{1m}(r,\tau-t) \\ G_{2m}(r,\tau-t) \\ G_{q+2,m}(r,\tau-t) \end{array}\right\} f_m(t)dt. \tag{5}$$

Here $f_m(t)$ are surface perturbations; $G_{ij}(r,\tau)$ ($\forall i,j=\overline{1,N+2}$) are the surface Green's functions determined from the solution of the following initial-boundary value problem:

$$\begin{gathered}
\frac{\partial^2 G_{1m}}{\partial \tau^2} = G''_{1m} + \frac{G'_{1m}}{r} - \frac{G_{1m}}{r^2} - bG'_{2m} - \sum_{j=1}^{N} \alpha_i G'_{j+2,m}, \\
\sum_{k=0}^{K} \frac{\tau_0^k}{k!} \frac{\partial^{k+1}}{\partial \tau^{k+1}} \left[G_{2m} + B\left(G'_{1m} + \frac{1}{r} G_{1m} \right) + \sum_{j=1}^{N} \beta_j G_{j+2,m} \right] = \kappa \left(G''_{2m} + \frac{1}{r} G'_{2m} \right), \\
\sum_{k=0}^{M} \frac{\tau_q^k}{k!} \frac{\partial^{k+1} G_{q+2,m}}{\partial \tau^{k+1}} = D_q \left(G''_{q+2,m} + \frac{G'_{q+2,m}}{r} \right) - \Lambda_q \left(G'''_{1m} + \frac{2}{r} G''_{1m} - \frac{G'_{1m}}{r^2} + \frac{G_{1m}}{r^3} \right) - M_q \left(G''_{2m} + \frac{G'_{2m}}{r} \right), \\
G_{1m}\Big|_{\tau=0} = 0, \quad \frac{\partial G_{1m}}{\partial \tau}\Big|_{\tau=0} = 0, \quad \frac{\partial^k G_{2m}}{\partial \tau^k}\Big|_{\tau=0} = 0, \quad \frac{\partial^m G_{q+2,m}}{\partial \tau^m}\Big|_{\tau=0} = 0, \\
G_{1m}\Big|_{r=1} = \delta_{1m}\delta(\tau), \quad G'_{2m}\Big|_{r=1} = \delta_{2m}\delta(\tau), \quad \left[\Lambda_q \left(G''_{1m} + \frac{G'_{1m}}{r} - \frac{G_{1m}}{r^2} \right) + M_q G'_{2m} - D_q G'_{q+2,m} \right]\Big|_{r=1} = \delta_{q+2,m}\delta(\tau),
\end{gathered} \tag{6}$$

where $\delta(\tau)$ is the Dirac delta function. Thus, the solution of problem (3), (4) reduces to determining the surface Green's functions.

Applying the Laplace transform with respect to time to problem (6) yields the following system of differential equations (s is the transform parameter, and the superscript L denotes the Laplace transform):

$$\begin{gathered}
s^2 G^L_{1m} = G''^L_{1m} + \frac{G'^L_{1m}}{r} - \frac{G^L_{1m}}{r^2} - bG'^L_{2m} - \sum_{j=1}^{N} \alpha_i G'^L_{j+2,m}, \\
\left(\sum_{k=0}^{K} \frac{\tau_0^k}{k!} s^{k+1} \right) \left[G^L_{2m} + B\left(G'^L_{1m} + \frac{1}{r} G^L_{1m} \right) + \sum_{j=1}^{N} \beta_j G^L_{j+2,m} \right] = \kappa \left(G''^L_{2m} + \frac{1}{r} G'^L_{2m} \right), \\
G^L_{q+2,m} \sum_{k=0}^{M} \frac{\tau_q^k}{k!} s^{k+1} = D_q \left(G''^L_{q+2,m} + \frac{G'^L_{q+2,m}}{r} \right) - \Lambda_q \left(G'''^L_{1m} + \frac{2}{r} G''^L_{1m} - \frac{G'^L_{1m}}{r^2} + \frac{G^L_{1m}}{r^3} \right) - M_q \left(G''^L_{2m} + \frac{G'^L_{2m}}{r} \right), \\
G^L_{1m}\Big|_{r=1} = \delta_{1m}, \quad G'^L_{2m}\Big|_{r=1} = \delta_{2m}, \quad \left[\Lambda_q \left(G''^L_{1m} + \frac{G'^L_{1m}}{r} - \frac{G^L_{1m}}{r^2} \right) + M_q G'^L_{2m} - D_q G'^L_{q+2,m} \right]\Big|_{r=1} = \delta_{q+2,m}.
\end{gathered} \tag{7}$$

For further transformations, the surface Green's functions are represented as Fourier series expansions in eigenfunctions:

$$G_{1m}(r,\tau) = \sum_{n=1}^{\infty} G_{1mn}(\tau) J_1(\lambda_n r), \quad G_{q+1,m}(r,\tau) = \sum_{n=0}^{\infty} G_{q+1,mn}(\tau) J_0(\lambda_n r) \qquad \left(q=\overline{1,N}\right), \tag{8}$$

where $J_\nu(z)$ are the Bessel functions of the first kind of order ν, and λ_n are the roots of the characteristic equation

$J_1(\lambda_n)=0$. The expansion coefficients $G^L_{qmn}(\tau)$ are sought in the form

$$G^L_{1mn}(s)=\frac{2}{J_0(\lambda_n)^2}\int\limits_0^1 rG^L_{1m}(r,s)J_1(\lambda_n r)dr,$$
$$G^L_{q+1,mn}(s)=\frac{2}{J_0(\lambda_n)^2}\int\limits_0^1 rG^L_{q+1,m}(r,s)J_0(\lambda_n r)dr,\qquad G^L_{q+1,m0}(s)=\int\limits_0^1 rG^L_{q+1,m}(r,s)dr. \tag{9}$$

Proceeding from this, the first equation of system (7) is multiplied by $rJ_1(\lambda_n r)$, whereas the second and third equations are multiplied by $rJ_0(\lambda_n r)$. After integration with respect to r over the interval from 0 to 1, and taking into account the boundary conditions and formulas (9), boundary value problem (7) is reduced to the following system of linear algebraic equations:

$$k_0(s)G^L_{2m0}(s)+k_0(s)\sum_{j=1}^N\beta_j G^L_{j+2,m0}(s)=-(Bk_0(s)\delta_{1m}-\kappa\delta_{2m}),\quad k_{q+2,0}(s)G^L_{q+2,m0}(s)=-\delta_{q+2,m}, \tag{10}$$

$$k_{1n}(s)G^L_{1mn}(s)-b\lambda_n G^L_{2mn}(s)-\lambda_n\sum_{j=1}^N\alpha_i G^L_{j+2,mn}(s)=-\frac{2\lambda_n\delta_{1m}}{J_0(\lambda_n)},$$
$$Bk_0(s)\lambda_n G^L_{1mn}(s)+k_{2n}(s)G^L_{2mn}(s)+k_0(s)\sum_{j=1}^N\beta_j G^L_{j+2,mn}(s)=-\frac{2(Bk_0(s)\delta_{1m}-\kappa\delta_{2m})}{J_0(\lambda_n)}, \tag{11}$$
$$\Lambda_q\lambda_n^3 G^L_{1mn}(s)+M_q\lambda_n^2 G^L_{2mn}(s)-k_{q+2,n}(s)G^L_{q+2,mn}(s)=-\frac{2(\Lambda_q\lambda_n^2\delta_{1m}-\delta_{q+2,m})}{J_0(\lambda_n)},$$

where

$$k_{1n}(s)=s^2+\lambda_n^2,\quad k_{2n}(s)=k_0(s)+\kappa\lambda_n^2,\quad k_0(s)=\sum_{k=0}^K\frac{\tau_0^k}{k!}s^{k+1},\quad k_{q+2,n}(s)=\sum_{k=0}^M\frac{\tau_q^k}{k!}s^{k+1}+\lambda_n^2 D_q.$$

From equalities (10) we determine $G^L_{2m0}(s)$ and $G^L_{q+2,m0}(s)$, whereas the solution of system (11) is found using Cramer's rule:

$$G^L_{2m0}(s)=-B\delta_{1m}+\frac{\kappa\delta_{2m}}{k_0(s)}+\sum_{j=1}^N\frac{\beta_j\delta_{j+2,m}}{k_{q+2,0}(s)},\quad G^L_{q+2,m0}(s)=-\frac{\delta_{q+2,m}}{k_{q+2,0}(s)},$$
$$G^L_{jmn}(s)=\frac{P_{jmn}(s)}{P_n(s)},\quad G^L_{q+2,mn}(s)=\frac{P_{q+2,mn}(s)}{Q_{qn}(s)}+\frac{2(\delta_{q+2,m}-\Lambda_q\lambda_n^2\delta_{1m})}{J_0(\lambda_n)k_{q+2,n}(s)},$$

where the polynomials $P_n(s)$ and $P_{imn}(s)$ have the form

$$P_n(s)=\big(k_{1n}k_{2n}+Bb\lambda_n^2k_0\big)\Pi_n-\lambda_n^6k_0\sum_{i=1}^N\Lambda_i\sum_{j=1}^N M_j\Pi_{ijn}+$$
$$+\lambda_n^2\sum_{j=1}^N\big[k_0\big(B\lambda_n^2\alpha_jM_j+b\lambda_n^2\beta_j\Lambda_j+k_{1n}\beta_jM_j\big)-k_{2n}\lambda_n^2\alpha_j\Lambda_j\big]\Pi_{jn},$$

$$P_{1mn}(s)=\frac{2}{J_0(\lambda_n)}\Bigg[(-\delta_{1m}\lambda_n(k_{2n}+Bbk_0)+\delta_{2m}\kappa b\lambda_n)\Pi_n+\sum_{j=1}^N\Big[\delta_{1m}\lambda_n^3(k_{2n}\alpha_j\Lambda_j-k_0[\beta_jM_j+B\alpha_jM_j+b\beta_j\Lambda_j])+$$
$$+\delta_{2m}\kappa\lambda_n^3\alpha_jM_j+\delta_{j+2,m}\lambda_n(k_0b\beta_j-k_{2n}\alpha_j)\Big]\Pi_{jn}-k_0\lambda_n^3\left(\delta_{1m}\lambda_n^2\sum_{i=1}^N\Lambda_i\sum_{j=1}^N M_j\Pi_{ijn}+\sum_{i=1}^N M_i\sum_{j=1}^N\delta_{j+2,m}\Pi_{ijn}\right)\Bigg],$$

$$P_{2mn}(s)=\frac{2}{J_0}\left[(\delta_{1m}k_0B(\lambda_n^2-k_{1n})+\delta_{2m}\kappa k_{1n})\Pi_n+\sum_{j=1}^{N}\Big[\delta_{1m}k_0\lambda_n^2(\lambda_n^2-k_{1n})\beta_j\Lambda_j-\delta_{2m}\kappa\lambda_n^4\alpha_j\Lambda_j+\right.$$

$$\left.+\delta_{j+2,m}k_0(k_{1n}\beta_j+B\lambda_n^2\alpha_j)\Big]\Pi_{jn}+k_0\lambda_n^4\left(\delta_{1m}\lambda_n^2\sum_{i=1}^{N}\Lambda_i\sum_{j=1}^{N}\Lambda_j\Pi_{ijn}+\sum_{i=1}^{N}\Lambda_i\sum_{j=1}^{N}\delta_{j+2,m}\Pi_{ijn}\right)\right],$$

$$P_{q+2,mn}(s)=\lambda_n^2(\Lambda_q\lambda_nP_{1mn}(s)+M_qP_{2mn}(s)),\quad Q_{qn}(s)=P_n(s)k_{q+2,n}(s),$$

$$\Pi_n=\prod_{q=1}^{N}k_{q+2,n}(s),\quad \Pi_{jn}=\prod_{q=1,q\neq j}^{N}k_{q+2,n}(s),\quad \Pi_{ijn}=(\alpha_i\beta_j-\alpha_j\beta_i)\prod_{q=1,q\neq i,j}^{N}k_{q+2,n}(s).$$

Taking into account that the functions $G^L_{qmn}(s)$ are rational functions of the parameter s, their originals are found using residues and tables of operational calculus [45] (the derivatives are taken with respect to the variable s):

$$G_{2m0}(\tau)=-B\delta_{1m}\delta(\tau)+\kappa\delta_{2m}\left(1+\sum_{k=1}^{K}\frac{\exp(s_{k0}\tau)}{k_0'(s_{k0})}\right)+\sum_{i=1}^{N}\beta_i\delta_{i+2,m}\left(1+\sum_{k=1}^{M}\frac{\exp(\chi_{ik0}\tau)}{k'_{i+2,0}(\chi_{ik0})}\right),$$

$$G_{q+2,m0}(\tau)=-\delta_{q+2,m}\left(1+\sum_{k=1}^{M}\frac{\exp(\chi_{qk0}\tau)}{k'_{q+2,0}(\chi_{qk0})}\right),$$

$$G_{jmn}(\tau)=\sum_{i=1}^{\Sigma}A^i_{jmn}\exp(s_{in}\tau),\quad A^i_{jmn}=\frac{P_{jmn}(s_{in})}{P_n'(s_{in})}\qquad (j=1,2,\ \Sigma=(M+1)N+K+3),\tag{12}$$

$$G_{q+2,mn}(\tau)=\sum_{i=1}^{\Sigma+M+1}A^i_{q+2,mn}\exp(s_{in}\tau)-\frac{2(\delta_{1m}\lambda_n^2\Lambda_q-\delta_{q+2,m})}{J_0(\lambda_n)}\sum_{i=1}^{M+1}\frac{\exp(\chi_{qin}\tau)}{k'_{q+2,n}(\chi_{qin})},\ A^i_{q+2,mn}=\frac{P_{q+2,mn}(s_{in})}{Q'_{qn}(s_{in})}.$$

Here s_{in} are the zeros of the polynomial $P_n(s)$, s_{i0} are the zeros of the polynomial $k_0(s)$, and χ_{qin} are the zeros of the polynomial $k_{q+2,n}(s)$.

Finally, substituting the Green's functions found from (8), (12) into the convolutions (5), we obtain the desired displacement functions and the temperature and concentration increments.

5. Examples

As a numerical example, consider a solid three-component ($N=2$) duralumin cylinder of radius $r=1$, in which component 1 is zinc (1%) and component 2 is copper (4.5%), diffusing in an aluminum matrix. The material is characterized by the following physical parameters [46]:

$$C_{12}=6.93\cdot10^{10}\,\mathrm{N/m^2},\quad C_{66}=2.56\cdot10^{10}\,\mathrm{N/m^2},\quad T_0=700\,\mathrm{K},\quad \rho=2700\,\mathrm{kg/m^3},\quad c_0=920\,\mathrm{J/(kg\cdot K)},$$
$$\alpha_1^{(1)}=6.55\cdot10^{7}\,\mathrm{J/kg},\quad \alpha_1^{(2)}=6.14\cdot10^{7}\,\mathrm{J/kg},\quad D_1^{(1)}=2.62\cdot10^{-12}\,\mathrm{m^2/s},\quad D_1^{(2)}=2.89\cdot10^{-14}\,\mathrm{m^2/s},$$
$$b_1=4.94\cdot10^{6}\,\mathrm{N/(K\cdot m^2)},\quad \kappa_1=240\,\mathrm{W/(m\cdot K)},\quad m^{(1)}=0.065\,\mathrm{kg/mol},\quad m^{(2)}=0.064\,\mathrm{kg/mol},$$
$$C_{11}=C_{22}=C_{12}+2C_{66},\ n_0^{(1)}=0.0084,\ n_0^{(2)}=0.045.$$

We assume that mechanical loads and mass transfer at the boundaries are absent ($f_1(\tau)=f_{q+2}(\tau)=0$). The relaxation times of the thermal and diffusion fluxes used in the calculations are indicated in the figure captions.

E x a m p l e 1. Surface perturbations are induced by nonstationary heat transfer according to the law

$$f_2(\tau)=H(\tau).\tag{13}$$

Substituting expressions (12), (13) into the convolutions (5), we obtain:

$$u(r,\tau)=\int_0^\tau G_{12}(r,\tau-t)f_2(t)dt=\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma}A_{12n}^{i}\frac{\exp(s_{in}\tau)-1}{s_{in}}J_1(\lambda_n r),$$

$$\vartheta(r,\tau)=\int_0^\tau G_{22}(r,\tau-t)f_2(t)dt=\kappa\left(\tau+\sum_{k=1}^{K}\frac{\exp(s_{k0}\tau)-1}{s_{k0}k_0'(s_{k0})}\right)+\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma}A_{22n}^{i}\frac{\exp(s_{in}\tau)-1}{s_{in}}J_0(\lambda_n r),$$

$$\eta_q(r,\tau)=\int_0^\tau G_{q+2,2}(r,\tau-t)f_2(t)dt=\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma+M+1}A_{q+2,2n}^{i}\frac{\exp(s_{in}\tau)-1}{s_{in}}J_0(\lambda_n r).$$

The computational results are shown in Figs. 1–5. One hundred terms of the Fourier series were used in the calculations. Further increasing the number of harmonics does not lead to noticeable changes in the results.

Figures 1, 2 present the displacement fields in the heated cylinder calculated using the model that accounts for the finite propagation speed of heat and mass transfer (Fig. 1*a*, 2*a*, solid and dotted lines in Fig. 2*b*) and using the classical model without relaxation (Fig. 1*b* and the dashed line in Fig. 2*b*). As can be seen, the wave nature of heat and mass propagation significantly affects the displacement field only at the initial time instants (a comparison of the plots in Figs. 1*a* and 1*b* shows that they differ by several orders of magnitude). As time progresses, these differences gradually diminish; for $\tau>8\cdot10^4$ they become insignificant, and for $\tau>2.5\cdot10^5$ the plots practically coincide (Fig. 2*b*).

Figures 3*a* and 3*b* show the temperature fields also calculated using different models (Fig. 3*a* with relaxation of thermal and diffusion fluxes taken into account, Fig. 3*b* without relaxation). For comparative analysis, these results are presented in Fig. 4*a*, where the dashed line corresponds to the solution obtained using the classical heat and mass transfer model without relaxation, whereas the remaining lines correspond to solutions for different relaxation times of thermal and diffusion fluxes. As in the case of displacements, the relaxation effects decay with time, and for $\tau>2\cdot10^4$ the models under consideration yield similar results.

The calculations used heat-flux relaxation times equal to 0.01 s (dotted lines in Figs. 2*b* and 4*a*) and 0.02 s (solid lines in Figs. 2*b* and 4*a*) [19, 20, 22, 23, 26], which, when converted to dimensionless quantities, are equal to $6.198\cdot10^3$ and $1.240\cdot10^4$, respectively. Thus, the threshold time values mentioned above, at which the effects caused by the finite propagation speed of thermal fluxes manifest themselves, are comparable with the relaxation times of these fluxes.

Based on the obtained numerical data, the following conclusion can be drawn: the displacement field induced by

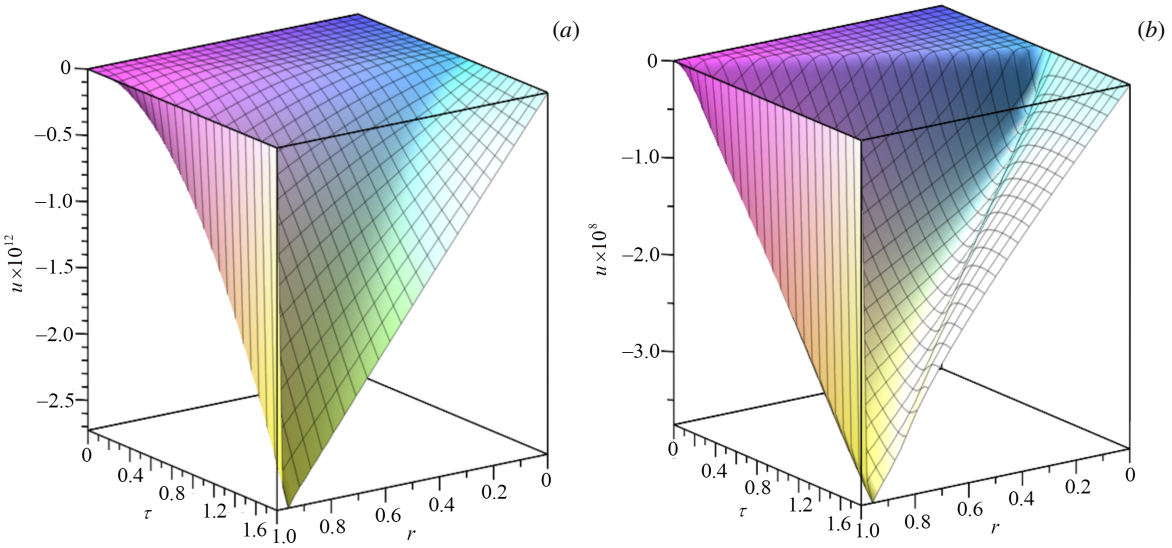


Fig. 1. Radial displacements $u(r,\tau)$ at small times with (*a*) and without (*b*) accounting for the finite propagation speed of heat and mass transfer

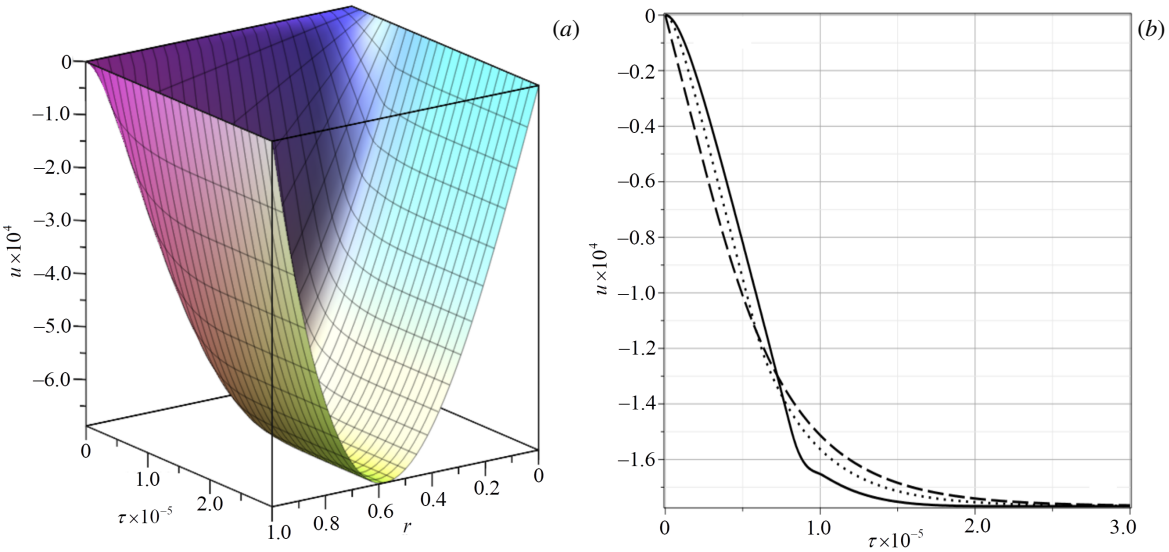


Fig. 2. Radial displacements $u(r,\tau)$ calculated at large times: space-time distribution (*a*) and at the point $r=0.1$ using different models (*b*); model without relaxation, $\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s (dashed line); model with relaxation for $\tau_\vartheta=0.01$ s,$\tau_\eta^{(q)}=0.1$ s (dotted line) and for $\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s (solid line)

cylinder heating weakly affects the heat and mass transfer process. Figure 4*b* presents, for comparison, the results obtained using the coupled thermomechanodiffusion model (3), (4) (solid and dashed lines) and the classical heat conduction model (dotted and dotted-dashed lines). It should also be noted here that the diffusion field arising due to heating has virtually no effect on either the displacement field or the temperature field; the plots in Figs. 1–4 are identical both in the presence and in the absence of diffusion.

At the same time, the temperature field itself initiates mass transfer of zinc and copper, as showing in Fig. 5. The character of mass transfer at the initial time instants, comparable with $\tau_\eta^{(q)}$, is significantly influenced by the inclusion of relaxation effects. It can be seen that the solid and dotted lines corresponding to the model with relaxation differ from the dashed line corresponding to the classical heat and mass transfer model. At the same time, the displacement field caused by cylinder heating no longer has practically any effect on mass transfer.

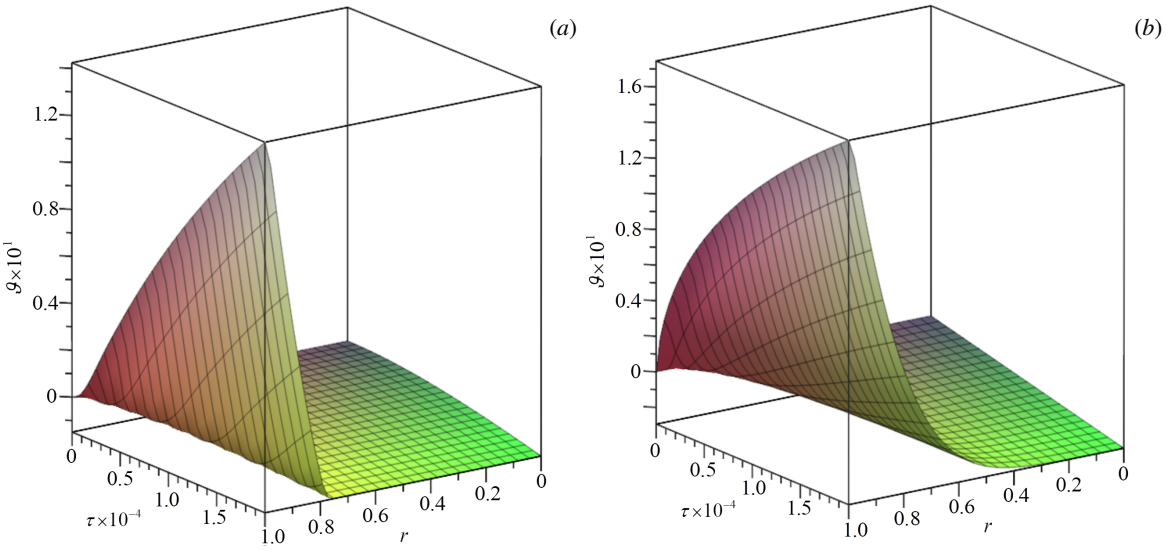


Fig. 3. Temperature field $\vartheta(r,\tau)$ with (*a*) and without (*b*) accounting for the finite propagation speed of thermal and diffusion fluxes

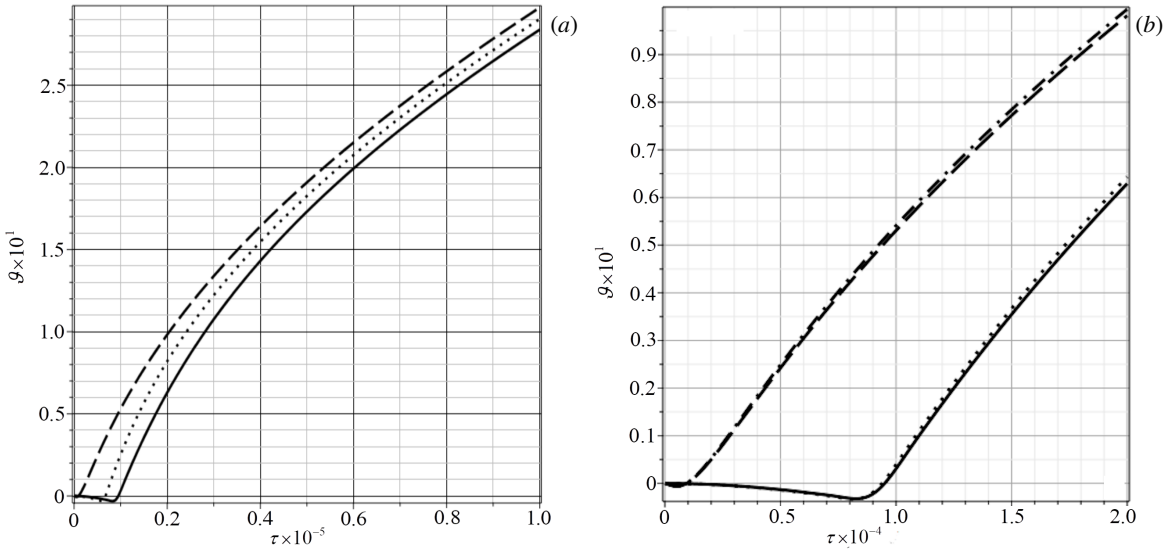


Fig. 4. Temperature field $\vartheta(r,\tau)$ computed using different models: model without relaxation ($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line), model with relaxation ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line, and $\tau_\vartheta=0.01$ s, $\tau_\eta^{(q)}=0.1$ s – dotted line) (*a*); coupled model ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line), uncoupled model ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – dotted line), coupled model without relaxation($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line), classical model (solution of the heat conduction problem without relaxation, $\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dotted-dashed line) (*b*)

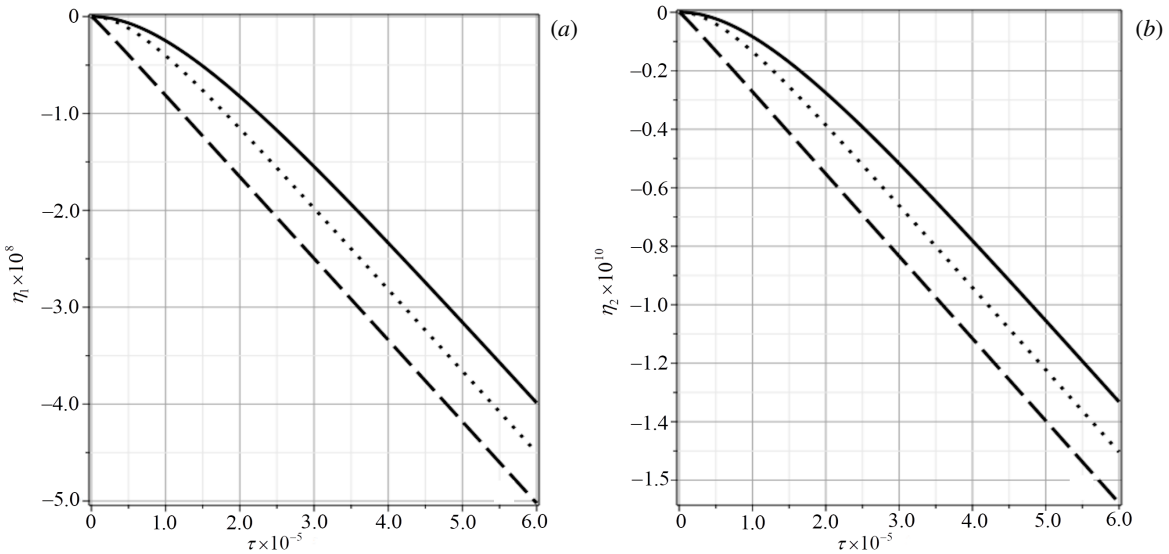


Fig. 5. Concentration increments of the 1st (*a*) and 2nd (*b*) components obtained using different models; model without relaxation ($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line), model with relaxation ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line, and $\tau_\vartheta=0.01$ s, $\tau_\eta^{(q)}=0.1$ s – dotted line)

E x a m p l e 2. Consider a cylinder subjected on its surface to an impulsive thermal load of the form

$$f_2(\tau)=\tau\exp(-\varepsilon\tau),\quad \varepsilon=0.01. \tag{14}$$

We also assume that mechanical loads and mass exchange sources at the boundaries are absent.

Substituting expressions (12), (14) into the convolutions (5), we obtain:

$$u(r,\tau)=\int_0^\tau G_{12}(r,\tau-t)f_2(t)dt=\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma}A_{12n}^i\frac{[\exp(s_{in}\tau)-(1+(\varepsilon+s_{in})\tau)\exp(-\varepsilon\tau)]}{(\varepsilon+s_{in})^2}J_1(\lambda_n r),$$

$$\vartheta(r,\tau)=\int_0^\tau G_{22}(r,\tau-t)f_2(t)dt=\kappa\left[\frac{1-(1+\varepsilon\tau)\exp(-\varepsilon\tau)}{\varepsilon^2}+\sum_{k=1}^{K}\frac{\exp(s_{k0}\tau)-(1+(\varepsilon+s_{k0})\tau)\exp(-\varepsilon\tau)}{k_0'(s_{k0})(\varepsilon+s_{k0})^2}\right]+$$

$$+\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma}A_{22n}^i\frac{[\exp(s_{in}\tau)-(1+(\varepsilon+s_{in})\tau)\exp(-\varepsilon\tau)]}{(\varepsilon+s_{in})^2}J_0(\lambda_n r),$$

$$\eta_q(r,\tau)=\int_0^\tau G_{q+2,2}(r,\tau-t)f_2(t)dt=\sum_{n=1}^{\infty}\sum_{i=1}^{\Sigma+M+1}A_{q+2,2n}^i\frac{[\exp(s_{in}\tau)-(1+(\varepsilon+s_{in})\tau)\exp(-\varepsilon\tau)]}{(\varepsilon+s_{in})^2}J_0(\lambda_n r).$$

The results obtained using different thermomechanodiffusion models are shown in Figs. 6, 7. In general, it may be noted that modifying the shape of the thermal pulse does not lead to a qualitative change in the character of interaction among the mechanical, thermal, and diffusion fields. Relaxation effects still manifest themselves significantly only at the initial stages of process evolution (Figs. 6, 7*a*, and 8), at times comparable with the relaxation times of the thermal and diffusion fluxes. As in the previous example, the feedback effect of the mechanical and diffusion fields on the temperature field is extremely small (Fig. 7*a*).

6. Conclusion

Thus, within the framework of a linear thermomechanodiffusion model accounting for finite propagation speeds of thermal and diffusion fluxes, the interaction of thermal, mechanical, and diffusion fields in deformable cylindrical bodies has been investigated, and the effect of field coupling on the stress-strain state of the structure under consideration has been demonstrated. Analysis of the obtained results makes it possible to draw the following conclusions:

1) Nonstationary surface heating initiates elastic deformations and mass transfer in the cylinder. At the same time, the displacements caused by heating are so small that they exert only a very slight feedback effect on the temperature field of the cylinder. As for diffusion processes, they also have virtually no influence on the temperature field.

2) The finite propagation speed of thermal and diffusion fluxes affects both the kinetics of heat and mass transfer and

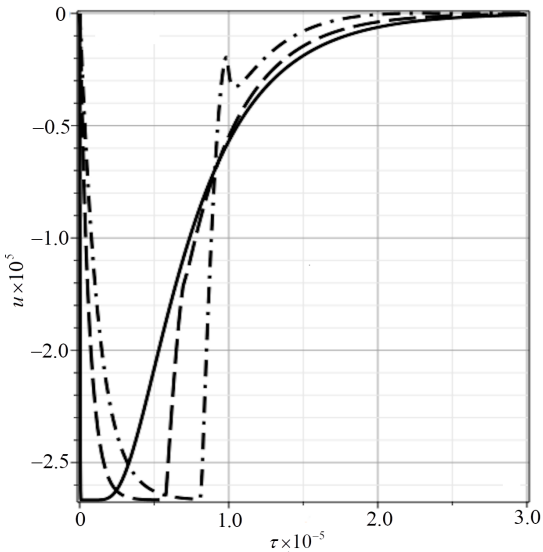


Fig. 6. Time dependences of displacements $u(r,\tau)$ at the point $r=0.1$: model without relaxation ($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line); model with relaxation ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line, and $\tau_\vartheta=0.01$ s, $\tau_\eta^{(q)}=0.1$ s – dotted line)

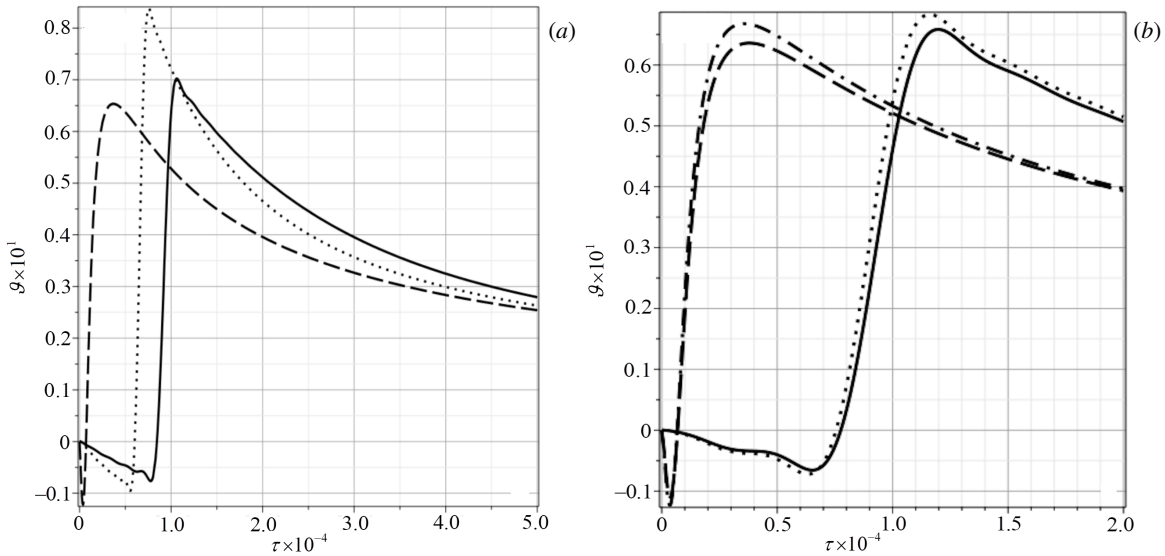


Fig. 7. Time dependences of temperature $\vartheta(r,\tau)$ at the point $r=0.9$: model without relaxation ($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line), model with relaxation ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line, and $\tau_\vartheta=0.01$ s, $\tau_\eta^{(q)}=0.1$ s – dotted line) (*a*); coupled model ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line), uncoupled model ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – dotted line), coupled model without relaxation ($\tau_\vartheta=0$s, $\tau_\eta^{(q)}=0$ s – dashed line), classical model (results of solving the heat conduction problem without relaxation, $\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s) (dotted-dashed line) (*b*)

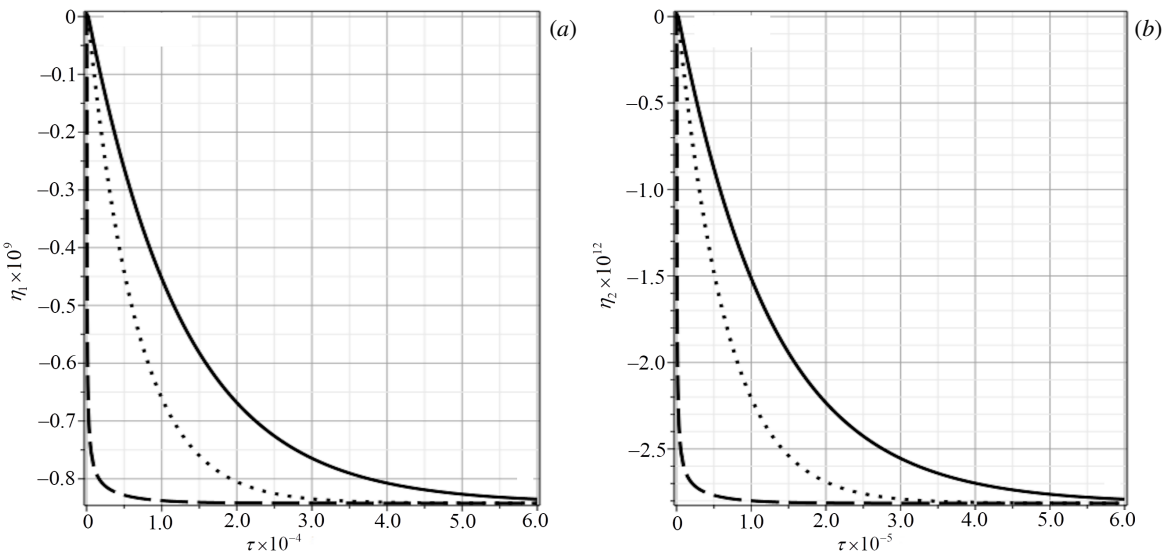


Fig. 8. Concentration increments of the 1st (*a*) and 2nd (*b*) components obtained using the model without relaxation ($\tau_\vartheta=0$ s, $\tau_\eta^{(q)}=0$ s – dashed line) and the model with relaxation ($\tau_\vartheta=0.02$ s, $\tau_\eta^{(q)}=0.2$ s – solid line, and $\tau_\vartheta=0.01$ s, $\tau_\eta^{(q)}=0.1$ s – dotted line)

the displacements inside the cylinder. Moreover, the factor of finite propagation speed of heat and mass manifests itself most significantly at the initial time instants comparable with the relaxation times, after which its importance vanishes.

3) The shape of the thermal pulse on the cylinder surface does not change the qualitative or quantitative character of the coupling of the mechanical, thermal, and diffusion fields. Two types of thermal loading are considered in this paper, and in both cases the field interaction is the same in the sense of the patterns described in items 1 and 2.

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