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Research article

An axisymmetric problem of gas filtration in a poroelastic medium

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The formulation and solution of the problem of burying carbon dioxide (carbon dioxide) in a poroelastic medium are considered. The model is based on the equations describing the filtration of liquids or gases in deformable porous media, which are a generalization of Muskett Leverett's models of poroelastic media. The assumption that the speed of movement of the solid skeleton of the medium is small made it possible to reduce the system of constitutive equations to two equations so that the effective pressure and porosity can be found. The gas filtration area refers to a rock formation in which an injection well is located at depth, and, on the sides, the formation is confined by impermeable rocks. The top of the formation coincides with the Earth surface and is permeable. The migration of carbon dioxide and its release to the surface occurs due to an increase in porosity at the top of the formation. Based on these assumptions, boundary conditions for the velocities of the gas and solid phases are set and then rewritten in terms of the desired function of the effective pressure of the medium. The resulting initial boundary value problem is solved numerically using a scheme of alternating directions and the fourth-order Runge–Kutta method. A difference scheme and an algorithm for solving the problem are given. The orders of uniform convergence in spatial and temporal variables were determined, and an approximate estimate for the rate of convergence of the numerical solution was obtained. Numerical modeling of several options for injecting carbon dioxide into the formation at different well depths and with different injection rates was carried out. Optimal gas injection conditions for its long-term geological storage were determined.

Keywords: porosity, poroelasticity, filtration, carbon dioxide gas, carbon dioxide, well, numerical solution, injection

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