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Research article

Fracture of brittle beams under antisymmetrical four-point bending

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The nucleation of cracks in structural elements during their service life is due to the degradation of the material or the presence of hidden defects. The structure therefore loses its original load-bearing capacity and fails under significantly lower external loads. As a rule, the fracture of structures due to crack growth occurs under mixed loading. In this paper, an eccentric beam of rectangular cross-section with an edge crack subjected to antisymmetric four-point loading is considered. By changing the position of the crack relative to the center of the beam, it is possible to obtain the entire range of mixed fracture modes: I+II, pure I and pure II modes. Using the finite element method, stress intensity coefficients for modes I and II of failure, as well as T-stresses, were obtained for different geometric parameters of the beam and different loading conditions. The length of the crack, its position relative to the beam center, and the length of the short span were varied. The methods commonly used for calculating T-stresses were analyzed. In the element closest to the crack tip, strong displacement oscillations, not mentioned in the literature, are observed, therefore to determine T-stresses with the highest possible accuracy it is suggested to calculate them from displacements, cutting off the 3–4 nodes closest to the crack tip. Experimental studies of the fracture toughness of ebonite in a mixed mode were carried out. For each type of loading and geometry, 3–5 identical specimens were tested. The tests were carried out under static load until the specimens were completely destroyed. In all experiments, the crack initiation angle and critical load were recorded. Six failure criteria were used to predict both failure direction and critical load: the generalized maximum tangential stress criterion, the extended maximum tangential strain criterion, the generalized strain energy density criterion, the generalized maximum averaged stress criterion, the maximum elastic energy release rate criterion and the generalized maximum elastic energy release rate criterion. The results obtained demonstrate good agreement between the experimental values of critical loads and the numerical calculations. The error in determining the crack initiation angle does not exceed 5%. Considering that the experimental results agree with the predictions of failure criteria for the beam subjected to antisymmetric four-point bend loading, this type of a beam specimen can be used to study mixed-type fracture in the elements manufactured from engineering materials, such as, for example, plexiglass, ebonite, and getinax.

Keywords: fracture mechanics, beam specimen, mixed mode loading, four-point bend, T-stress, finite element analysis

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References

1. *Suresh S.*, *Shih C.F.*, *Morrone A.*, *O'Dowd N.P.* Mixed-mode fracture toughness of ceramic materials. Journal of the American Ceramic Society. 1990. Vol. 73, no. 5. P. 1257–1267. DOI: 10.1111/j.1151-2916.1990.tb05189.x
2. *Fett T.*, *Gerteisen G.*, *Hahnenberger S.*, *Martin G.*, *Munz D.* Fracture tests for ceramics under mode-I, mode-II and mixed-mode loading. Journal of the European Ceramic Society. 1995. Vol. 15, no. 4. P. 307–312. DOI: 10.1016/0955-2219(95)90353-K
3. *Tikare V.*, *Choi S.R.* Combined Mode I–Mode II Fracture of 12-mol%-Ceria-Doped Tetragonal Zirconia Polycrystalline Ceramic. Journal of the American Ceramic Society. 1997. Vol. 80, no. 6. P. 1624–1626. DOI: 10.1111/j.1151-2916.1997.tb03030.x
4. *Choi S.R.*, *Zhu D.*, *Miller R.A.* Fracture behavior under mixed-mode loading of ceramic plasma-sprayed thermal barrier coatings at ambient and elevated temperatures. Engineering Fracture Mechanics. 2005. Vol. 72, no. 13. P. 2144–2158. DOI: 10.1016/j.engfracmech.2005.01.010
5. *Aliha M.R.M.*, *Ayatollahi M.R.* Analysis of fracture initiation angle in some cracked ceramics using the generalized maximum tangential stress criterion. International Journal of Solids and Structures. 2012. Vol. 49. P. 1877–1883. DOI: 10.1016/j.ijsolstr.2012.03.029
6. *Li M.*, *Sakai M.* Mixed-Mode Fracture of Ceramics in Asymmetric Four-Point Bending: Effect of Crack-Face Grain Interlocking/Bridging. Journal of the American Ceramic Society. 1996. Vol. 79, no. 10. P. 2718–2726. DOI: 10.1111/j.1151-2916.1996.tb09037.x
7. *Ayatollahi M.R.*, *Aliha M.R.M.* Mixed mode fracture in soda lime glass analyzed by using the generalized MTS criterion. International Journal of Solids and Structures. 2009. Vol. 46, no. 2. P. 311–321. DOI: 10.1016/j.ijsolstr.2008.08.035
8. *Aliha M.R.M.*, *Ayatollahi M.R.* Geometry effects on fracture behaviour of polymethyl methacrylate. Materials Science and Engineering: A. 2010. Vol. 527, no. 3. P. 526–530. DOI: 10.1016/j.msea.2009.08.055
9. *Araki W.*, *Nemoto K.*, *Adachi T.*, *Yamaji A.* Fracture toughness for mixed mode I/II of epoxy resin. Acta Materialia. 2005. Vol. 53, no. 3. P. 869–875. DOI: 10.1016/j.actamat.2004.10.035
10. *He M.Y.*, *Hutchinson J.W.* Asymmetric Four-Point Crack Specimen. Journal of Applied Mechanics. 2000. Vol. 67, no. 1. P. 207–209. DOI: 10.1115/1.321168
11. *Maccagno T.M.*, *Knott J.F.* The fracture behaviour of PMMA in mixed modes I and II. Engineering Fracture Mechanics. 1989. Vol. 34, no. 1. P. 65–86. DOI: 10.1016/0013-7944(89)90243-9
12. *Ayatollahi M.R.*, *Shadlou S.*, *Shokrieh M.M.* Mixed mode brittle fracture in epoxy/multi-walled carbon nanotube nanocomposites. Engineering Fracture Mechanics. 2011. Vol. 78, no. 14. P. 2620–2632. DOI: 10.1016/j.engfracmech.2011.06.021
13. *Aliha M.R.*, *Ayatollahi M.R.*, *Smith D.J.*, *Pavier M.J.* Geometry and size effects on fracture trajectory in a limestone rock under mixed mode loading. Engineering Fracture Mechanics. 2010. Vol. 77, no. 11. P. 2200–2212. DOI: 10.1016/j.engfracmech.2010.03.009

14. *Erarslan N.*, *Williams D.J.* Mixed-Mode Fracturing of Rocks Under Static and Cyclic Loading. Rock Mechanics and Rock Engineering. 2013. Vol. 46. P. 1035–1052. DOI: 10.1007/s00603-012-0303-5
15. *Aliha M.R.M.*, *Hosseinpour G.R.*, *Ayatollahi M.R.* Application of Cracked Triangular Specimen Subjected to Three-Point Bending for Investigating Fracture Behavior of Rock Materials. Rock Mechanics and Rock Engineering. 2013. Vol. 46. P. 1023–1034. DOI: 10.1007/s00603-012-0325-z
16. *Wang C.*, *Zhu Z.M.*, *Liu H.J.* On the I–II mixed mode fracture of granite using four-point bend specimen. Fatigue & Fracture of Engineering Materials & Structures. 2016. Vol. 39, no. 10. P. 1193–1203. DOI: 10.1111/ffe.12422
17. *Ayatollahi M.R.*, *Aliha M.R.M.* Mixed mode fracture analysis of polycrystalline graphite – A modified MTS criterion. Carbon. 2008. Vol. 46, no. 10. P. 1302–1308. DOI: 10.1016/j.carbon.2008.05.008
18. *Mirsayar M.M.*, *Berto F.*, *Aliha M.R.M.*, *Park P.* Strain-based criteria for mixed-mode fracture of polycrystalline graphite. Engineering Fracture Mechanics. 2016. Vol. 156. P. 114–123. DOI: 10.1016/j.engfracmech.2016.02.011
19. *Efimov V.* Brazilian tensile strength testing. Journal of Mining Science. 2021. Vol. 57, no. 6. P. 922–932. DOI: 10.1134/S1062739121060053
20. *Ayatollahi M.R.*, *Aliha M.R.M.* Wide range data for crack tip parameters in two disc-type specimens under mixed mode loading. Computational Materials Science. 2007. Vol. 38. P. 660–670. DOI: 10.1016/j.commatsci.2006.04.008
21. *Torabi A.R.*, *Etesam S.*, *Sapora A.*, *Cornetti P.* Size effects on brittle fracture of Brazilian disk samples containing a circular hole. Engineering Fracture Mechanics. 2017. Vol. 186. P. 496–503. DOI: 10.1016/j.engfracmech.2017.11.008
22. *Aliha M.R.M.*, *Ayatollahi M.R.*, *Pakzad R.* Brittle Fracture Analysis Using a Ring-Shape Specimen Containing Two Angled Cracks. International Journal of Fracture. 2008. No. 1. P. 63–68. DOI: 10.1007/s10704-008-9280-9
23. *Mirsayar M.M.*, *Park P.* The role of T-stress on kinking angle of interface cracks. Materials & Design. 2015. Vol. 80. P. 12–19. DOI: 10.1016/j.matdes.2015.05.007
24. *Lin Q.*, *Bian X.*, *Pan P.-Z.*, *Gao Y.*, *Lu Y.* Criterion of local symmetry visualized in small eccentric single edge notched bend (E-SENB) rock specimens. Engineering Fracture Mechanics. 2021. Vol. 248. 107709. DOI: 10.1016/j.engfracmech.2021.107709
25. *Shahani A.R.*, *Tabatabaei S.A.* Effect of T-stress on the fracture of a four point bend specimen. Materials & Design. 2009. Vol. 30, no. 7. P. 2630–2635. DOI: 10.1016/j.matdes.2008.10.031
26. *Li Y.*, *Dong S.*, *Pavie M.J.* Measurement of the mixed mode fracture strength of green sandstone using three-point bending specimens. Geomechanics and Engineering. 2020. Vol. 20, no. 1. P. 9–18. DOI: 10.12989/gae.2020.20.1.009
27. *Williams M.L.* On the stress distribution at the base of a stationary crack. Journal of Applied Mechanics. 1957. Vol. 24. P. 109–114.
28. *Kurguzov V.*, *Demeshkin A.*, *Kuznetsov D.* Three-point bending of eccentric edge crack specimens under mixed mode loading. Computational Continuum Mechanics. 2023. Vol. 16, no. 3. P. 345–357. DOI: 10.7242/1999-6691/2023.16.3.29
29. MARC 2020. Volume A: Theory and User Information. Santa Ana (CA): MSC.Software Corporation, 2020. 1061 p.
30. *Shih C.F.*, *Asaro R.J.* Elastic-Plastic Analysis of Cracks on Bimaterial Interfaces: Part I—Small Scale Yielding. Journal of Applied Mechanics. 1988. Vol. 55, no. 2. P. 299–316. DOI: 10.1115/1.3173676
31. *Gupta M.*, *Alderliesten R.C.*, *Benedictus R.* A review of T-stress and its effects in fracture mechanics. Engineering Fracture Mechanics. 2015. Vol. 134. P. 218–241. DOI: 10.1016/j.engfracmech.2014.10.013
32. *Tyrymov A.* Numerical simulation of T-stresses and stress biaxiality factor for a centrally cracked specimen under mixed boundary conditions. Computational continuum mechanics. 2020. Vol. 13, no. 4. P. 393–401. DOI: 10.7242/1999-6691/2020.13.4.30
33. *Stepanova L.*, *Frolov K.* Finite element study of mixed mode loading of the cracked semicircular disc under bending. PNRPU Mechanics Bulletin. 2018. No. 3. P. 128–137. DOI: 10.15593/perm.mech/2018.3.12
34. *Chang K.J.* On the maximum strain criterion – a new approach to the angled crack problem. Engineering Fracture Mechanics. 1981. Vol. 14. P. 107–124. DOI: 10.1016/0013-7944(81)90021-7
35. *Wu H.C.* Dual failure criterion for plain concrete. Journal of Engineering Mechanics–ASCE. 1974. Vol. 100, no. 6. P. 1167–1181.
36. *Lazzarin P.*, *Zambardi R.* A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches. International Journal of Fracture. 2001. P. 275–298. DOI: 10.1023/A:1013595930617
37. *Lazzarin P.*, *Livieri P.*, *Berto F.*, *Zappalorto M.* Local strain energy density and fatigue strength of welded joints under uniaxial and multiaxial loading. Engineering Fracture Mechanics. 2008. Vol. 75. P. 1875–1889. DOI: 10.1016/J.ENGFRACMECH.2006.10.019
38. *Foti P.*, *Santonocito D.*, *Risitano G.*, *Berto F.* Fatigue assessment of cruciform joints: Comparison between Strain Energy Density predictions and current standards and recommendations. Engineering Structures. 2021. Vol. 230. 111708. DOI: 10.1016/j.engstruct.2020.111708
39. *Kusch A.*, *Salamina S.*, *Crivelli D.*, *Berto F.* Strain Energy Density as Failure Criterion for Quasi-Static Uni-axial Tensile Loading. Frattura ed Integrità Strutturale. 2021. Vol. 15. P. 331–349. DOI: 10.3221/IGF-ESIS.57.24
40. *Sajjadi S.H.*, *Khorshidvand A.R.*, *Jabbari M.*, *Javadi M.* Comparing Energy-Based Criterion with Strain-Based Criterion for Predicting Brittle Fracture in V-Shaped Notches under Mixed-Mode I/II Loading Conditions. Physical Mesomechanics. 2021. Vol. 24, no. 3. P. 291–302. DOI: 10.1134/S1029959921030073

41. *Ayatollahi M.R.*, *Rashidi Moghaddam M.*, *Berto F.* A generalized strain energy density criterion for mixed mode fracture analysis in brittle and quasi-brittle materials. Theoretical and Applied Fracture Mechanics. 2015. Vol. 79. P. 70–76. DOI: 10.1016/j.tafmec.2015.09.004

42. *Ayatollahi M.R.*, *Rashidi Moghaddam M.*, *Razavi N.*, *Berto F.* Geometry effects on fracture trajectory of PMMA samples under pure mode-I loading. Engineering Fracture Mechanics. 2016. Vol. 163. P. 449–461. DOI: 10.1016/j.engfracmech.2016.05.014

43. *Kurguzov V.*, *Kuznetsov D.* Fracture analysis of Brazilian circular hole disk under mixed mode loading. Engineering Fracture Mechanics. 2024. Vol. 297. 109875. DOI: 10.1016/j.engfracmech.2024.109875

44. *Matvienko Y.G.* Maximum Average Tangential Stress Criterion for Prediction of the Crack Path. International Journal of Fracture. 2012. Vol. 176. P. 113–118. DOI: 10.1007/s10704-012-9715-1

45. *Wieghardt K.* Uber das Spalten und Zerreisen elastischer Korper. Zeitschrift fur Mathematik und Physik. 1907. Vol. 55, no. 1/2. P. 60–103.

46. *Neuber H.* Kerbspannungslehre, Grundlagen fur eine genaue Spannungsrechnung. Berlin: Springer-Verlag, 1937. 160 p.

47. *Novozhilov V.V.* On a necessary and sufficient criterion for brittle strength. Journal of Applied Mathematics and Mechanics. 1969. Vol. 33, no. 2. P. 201–210. DOI: 10.1016/0021-8928(69)90025-2

48. *Dyskin A.V.* Crack growth criteria incorporating non-singular stresses: size effect in apparent fracture toughness. International Journal of Fracture. 1997. Vol. 83. P. 191–206. DOI: 10.1023/A:1007304015524

49. *Matvienko Y.G.* Local fracture criterion to describe failure assessment diagrams for a body with a crack/notch. International Journal of Fracture. 2003. Vol. 124. P. 107–112. DOI: 10.1023/B:FRAC.0000018229.08344.96

50. *Matvienko Y.G.*, *Semenova M.M.* The concept of the average stress in the fracture process zone for the search of the crack path. Frattura ed Integrità Strutturale. 2015. Vol. 9, no. 34. P. 255–260. DOI: 10.3221/IGF-ESIS.34.27

51. *Hussain M.A.*, *Pu S.L.*, *Underwood J.* Strain Energy Release Rate for a Crack Under Combined Mode I and Mode II. Fracture Analysis. ASTM STP 560. 1974. P. 2–28. DOI: 10.1520/STP33130S

52. *Nuismer R.J.* An energy release rate criterion for mixed mode fracture. International Journal of Fracture. 1975. Vol. 11. P. 245–250. DOI: 10.1007/BF00038891

53. *Hou C.*, *Jin X.*, *Fan X.*, *Xu R.*, *Wang Z.* A generalized maximum energy release rate criterion for mixed mode fracture analysis of brittle and quasi-brittle materials. Theoretical and Applied Fracture Mechanics. 2019. Vol. 100. P. 78–85. DOI: 10.1016/j.tafmec.2018.12.015