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Research article

# Modeling of turbulent natural convection based on a two-fluid approach

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The paper provides mathematical modeling of turbulent natural air convection at a heated vertical plate based on a fairly recently developed two-liquid turbulence model. The considered problem, despite its relative simplicity, contains all the main elements characteristic of the currents near the wall due to buoyancy forces. A significant disadvantage of the RANS turbulence models used to solve such problems is that for their numerical implementation it is necessary to set the laminar-to-turbulent transition point, which must be determined experimentally. Thus, all RANS models are unable to describe a laminar-to-turbulent transition zone. Therefore, the main purpose of the work is to test the ability of the two-liquid turbulence model to describe the transition zone. Well-known publications have shown that the two-liquid model has high accuracy and stability, and is also able to adequately describe anisotropic turbulence. The turbulence model used in this work is supplemented with an additional thermal force, which can be ignored in many flows with forced convection. However, in the natural convection currents, it is this force that contributes to the transition of the flow regime. To validate the model, as well as to verify the computational procedure, the numerical results obtained are compared with the results of the well-known RANS turbulence models (the one-parameter Spalart-Allmaras (SA) model and the Reynolds stress transfer (RSM) model), as well as with the available experimental data. It is shown that the two-liquid model adequately reproduces the laminar-to-turbulent transition zone, and the numerical results obtained are in good agreement with experimental data.

*Keywords:* two-fluid approach to turbulence, heated vertical plate, verification, natural convection, thermal force, Nusselt number, transition zone

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## 1. Introduction

Heat transfer in turbulent natural convective flows is of great interest both in theoretical and applied aspects. As an example, it is sufficient to cite natural phenomena where turbulent convection plays a decisive role [1]. Convection processes are also significant in industry. For example, natural convection heat transfer serves as a reliable and economical method of cooling in the rapidly growing electronics industry, where hundreds of modules can be placed on a small base. Since the density of these fuel elements is constantly increasing, the emitted heat must be efficiently dissipated not only to protect them, but also to increase their lifetime. It is often necessary to cool the interior surfaces of vertical open ducts by natural convection where heat transfer rates are low. Thus, knowledge of the behavior of natural convection flow in enclosed spaces is not only useful but also necessary, especially in heat transfer fluid systems used in various nuclear and solar power applications [2].

Because of its importance, the problem of natural convection has attracted increasing attention of researchers in recent years [3]. The existing convective processes can be divided into free and forced convection. The authors of most publications adhere to this gradation [4, 5]. Many scientists have studied the issues of free and forced convection; in particular, the description of convective systems is given in detail in [6–8]. Natural convection can be either laminar or turbulent. In natural conditions, as well as in large spaces, convection is turbulent. Therefore, modeling turbulence — one of the problems in the study of convection processes. In recently, along with laboratory tests, numerical experiment — study of phenomena using Computational Fluid Dynamics (CFD) methods has been used. Modern program complexes allow the investigation of a large number of hydrodynamic, heat and mass-exchange processes [9–12]. The most common complexes are ANSYS, COMSOL Multyphysics, FlowVision and others, which contain various turbulence models and are designed to study the majority of flows occurring in nature and in man-made technologies that have a turbulent character. Therefore, the application of software packages to solve a particular fluid dynamics problem poses a challenge to scientists in selecting an appropriate turbulence model. There are basically three approaches to mathematical modeling of turbulence. The first approach is based on Direct Numerical Simulation (DNS), which is based on the hypothesis that the Navier-Stokes equations are sufficient to describe turbulent flows. A nonstationary system of Navier-Stokes equations

in three-dimensional formulation is numerically solved. For integration, computational cells with dimensions smaller than the Kolmogorov scale are required. The DNS approach requires a large amount of computational resources, and its use for calculation of complex hydrodynamic problems is not possible at the present time. In the second approach, the Reynolds Averaged Navier-Stokes (RANS) equations are averaged over the Reynolds number, which leads to a nonclosed system of equations. Therefore, turbulence models are additionally introduced to close this system. All of them are called RANS-models. The advantage of RANS models is that their numerical realization requires significantly less computational resources compared to DNS methods [13]. For this reason, the RANS approach finds the widest application in solving engineering problems. Despite the significant progress achieved in the field of modeling turbulent flows on the basis of RANS models, there are still unsolved problems related to the calculation of dynamics and heat transfer in turbulent natural convective flows. As shown in [14, 15], the use of the most common two-parameter semiempirical turbulence models of the type  $k - \varepsilon$  for solving applied problems of convective heat transfer does not provide the accuracy of description of naturally convective turbulent flows necessary for practice. In this connection, attempts to improve the existing turbulence models and to construct new turbulence models for this class of flows have been made for many years. In many works, the main emphasis is placed on the need to directly account for the influence of buoyancy effects in traditional turbulence models (e.g., in  $k - \varepsilon$ -models) by including additional terms in the corresponding transfer equations of turbulent flow characteristics [16, 17]. However, such a method leads both to the introduction and determination of a number of additional empirical constants and to an increase in the number of differential equations included in the model. This, in turn, further complicates the already computationally laborious low-Reynolds versions of  $k - \varepsilon$ -models and other similar models.

Another direction in modeling natural convection is the use of more complex RANS models, in particular, Reynolds stress transport models (RSMs). However, because of their complexity, their application is limited mainly to the solution of rather simple model problems. The extensive numerical studies and parametric calculations necessary for solving practical problems of convective heat transfer are not always justified and efficient. In recent years, models that are substantially superior to the traditional models in terms of both computational efficiency and accuracy in solving the range of problems of forced convection have gained wide popularity. In particular, these include the one-parameter model based on the solution of the turbulent viscosity transport equation, — the Spalart-Allmaras (SA) model [18], and the two-parameter Menter (SSG) model [19], which is a combination of the standard  $k - \varepsilon$ -model and the well-known  $k - \omega$ -Wilcox model [20]. The capabilities of these models and their substantial superiority over the standard  $k - \varepsilon$ -models as applied to solving a number of complex problems of forced convection have been demonstrated in many papers. Thus, in [21] these models are used to solve some problems of natural convection and it is noted that they give more accurate solutions than complex models. However, in the opinion of the authors of this paper, a significant disadvantage of almost all RANS models is that they are unable to represent the transition from laminar to turbulent flow when solving natural convection problems. For example, in the above-mentioned work, for the calculation of turbulent convection near a heated vertical plate, the zone of flow transition from the laminar regime to the turbulent regime in all the models considered was set on the basis of experimental studies. The third approach to mathematical modeling of turbulence is the so-called 2-fluid approach proposed by Spalding [22, 23]. It has been used to study: the Rayleigh-Taylor instability [24]; the combustion process [25]; turbulent stratified flows [26]; confined and free turbulent shear flows [27], and others. The 2-fluid approach was further developed in work [28, 29], where its ability to describe with much accuracy the turbulent processes of momentum and heat transport in anisotropic turbulent flows is presented. The purpose of the present work is to demonstrate the feasibility of the 2-liquid approach to account for natural turbulent air convection near a heated vertical plate. Despite its relative simplicity, the chosen problem contains all the main elements characteristic of currents near a wall due to buoyancy forces. Along with reliable experimental data [30, 31] for the flow under consideration, the results of calculations using the SA and RSM models from [21] are also presented.

#### 2. System of equations for turbulent natural convection near a heated vertical infinite plate in the 2-liquid approach

It was shown in [28] that turbulent flow can be represented as a heterogeneous mixture of two fluids not only with different velocities but also with different temperatures:

$$V_{1i} = V_i + \vartheta_i, \quad V_{2i} = V_i - \vartheta_i, \quad T_1 = T + t, \quad T_2 = T - t.$$
(1)

In expressions (1) denote:  $V_i, T$  — averaged velocity and temperature of the turbulent flow;  $\vartheta_i, t$  — relative velocity and temperature of the heterogeneous mixture. The derivation of the equations of turbulent thermodynamics is given

in the aforementioned paper in detail. Based on these equations, we give a system of equations for convective flow of air with velocity  $V_0$  near a vertical plate of length L heated to temperature  $T_w$  (Fig. 1):

$$\begin{aligned} \frac{\partial \rho U}{\partial x} + \frac{\partial \rho V}{\partial y} &= 0, \\ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} &= \frac{\partial}{\partial y} \left( \nu \frac{\partial U}{\partial y} - u \vartheta \right) + g \frac{T - T_0}{T_0}, \\ U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} &= -(1 - C_s) \vartheta \frac{\partial U}{\partial y} + \frac{\partial}{\rho \partial y} \left( \rho \nu_{xy} \frac{\partial u}{\partial y} \right) - K_f u + g \frac{t}{T_0}, \\ U \frac{\partial \vartheta}{\partial x} + V \frac{\partial \vartheta}{\partial y} &= -C_s u \frac{\partial U}{\partial y} + \frac{\partial}{\rho \partial y} \left( 2\rho \nu_{yy} \frac{\partial \nu}{\partial y} \right) - K_f \vartheta - \vartheta^2_T \frac{t}{T_0^2} \frac{\partial T}{\partial y}, \\ U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} &= \frac{\partial}{\rho \partial y} \left( \rho k \frac{\partial T}{\partial y} - \rho t \vartheta \right), \\ U \frac{\partial t}{\partial x} + V \frac{\partial t}{\partial y} &= -\frac{\partial T}{\partial y} \vartheta + \frac{\partial}{\rho \partial y} \rho k_y \frac{\partial t}{\partial y} - K_t t, \\ \nu_{xy} &= 3\nu + 2 \left| \frac{u \vartheta}{\partial U / \partial y} \right|, \quad \nu_{yy} = 3\nu + 2 \left| \frac{\vartheta \vartheta}{\partial U / \partial y} \right|, \quad k_y = 3k + 2 \left| \frac{t \vartheta}{\partial T / \partial y} \right|, \\ \rho T &= \rho_0 T_0. \end{aligned}$$

Here: x — coordinate pointing along the vertical plate; y — coordinate perpendicular to the plate; U,V — longitudinal and transverse velocities;  $u,\vartheta$  — longitudinal and transverse relative velocities; T — air temperature;  $T_0$  — air temperature away from the plate;  $\rho$  — density of air;  $\rho_0$  — density of medium away from the plate;  $\nu$  — kinematic viscosity; k — air diffusivity;  $\nu_{xy}, \nu_{yy}$  — molar viscosities;  $k_y$  — molar diffusivity; g — free-fall acceleration;  $C_s$  — correction factor;  $K_f$  and  $K_t$  — friction and heat transfer coefficients; t — nefluctuating temperature. The pressure has a constant value everywhere. In view of its smallness, we neglect the longitudinal derivatives in the diffusion terms of the right-hand sides of equations (2). As can be seen, in the fourth equation, an additional thermal force is introduced into the right-hand side of the equation

$$f_T = -\rho \vartheta_T^2 \frac{t}{T_0^2} \frac{\partial T}{\partial y}.$$
(3)

In (3), the parameter  $\vartheta_T$  has the dimension of velocity, and by numerical investigation it is found to be equal to  $\vartheta_T = 2.05$  m/s. In many turbulent flows, the characteristic velocity  $V_0$  is such that  $\vartheta_T/V_0 \ll 1$ . Therefore, in problems with forced convection, the force  $f_T$  may be substantially smaller than other forces, hence this force can be dis counted. However, in natural convection, the flow velocity is small and in these cases the thermal force cannot be neglected. It will be shown below that in natural convective flow near a heated vertical plate, the transition of laminar flow to turbulent flow is due to this force because the flow velocity is small in the transition zone.

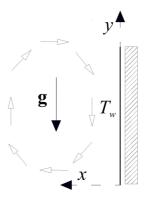


Fig. 1. Schematic to task

We reduce the system of equations (2) to dimensional form. For this purpose, we take all velocities to a quantity with velocity dimension  $\sqrt{gL(T_w - T_0)/T_0}$ , all distances — to length L = 1 m, all temperatures — to the ambient temperature  $T_0 = 289$  K, and the air density — to the ambient density  $\rho_0 = 1.23$  kg/m<sup>3</sup>. The temperature of the heated plate is  $T_0 = 333$  K. For numerical realization of the system (2), it is convenient to introduce Mises variables  $\xi$  and  $\psi$  [32, 33]:

$$\xi \!=\! x, \quad \rho U \!=\! \frac{\psi \partial \psi}{\partial y}, \quad \rho V \!=\! -\frac{\psi \partial \psi}{\partial x},$$

In new variables, the first equation in (2) — the continuity equation, is satisfied automatically. We numerically solve the system (2) on the basis of an implicit finite-difference scheme. In the transverse direction, we will use central differences. To solve the implicit scheme, we apply the run method. We take the steps of integration over the Mises variables to be  $\Delta \xi = 0.00002$  and  $\Delta \psi = 0.001$ , and let the number of computational points in the transverse direction be 1000. Note that the computational experiments allowed us to conclude that there is no significant influence of the mesh parameters on the result. To close the system of solving equations (2), we set the sticking conditions on the plate, and equate all velocities away from it to zero. At entry, that is, at  $\xi = 0$ , we set the dimensionless relative velocities as follows:  $\bar{u} = 10^{-20}$ ,  $\bar{\vartheta} = 0$ . To calculate air viscosity more accurately, we use Sutherland's law:

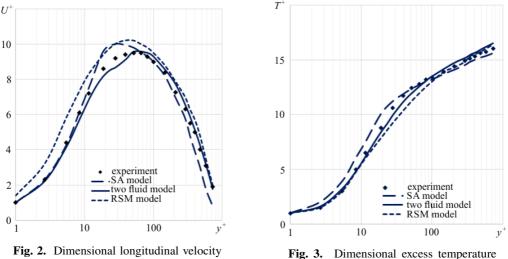
$$\frac{v}{v_0} = \left(\frac{T}{T_0}\right)^{2.5} \frac{T_0 + 110.4}{T + 110.4}.$$
(4)

### 3. Discussion of results

profile

Figures 2–6 below summarize the numerical results calculated from the 2-liquid model. For comparison, these figures also contain results from [21] corresponding to the SA and RSM turbulence models and experimental data from [30, 31]. The drawing 2 shows the longitudinal velocity profile in section  $\xi = 2.535$ . Here  $U^+ = U/u_*$ , where  $u_* = \sqrt{\nu(\partial U/\partial y)}|_w$  — dynamic velocity near the wall and  $y^+ = \nu y/u_*$ . It can be seen from the figure that the SA model describes well the longitudinal velocity profile near the plate, and the RSM model — far from here. Results closer to the experimental data are obtained by the 2-liquid model.

The drawing 3 shows the profile of the dimensionless excess temperature in section  $\xi = 2.535$ . Here  $T^+ = (T_w - T)/t_*$ , where  $t_* = [\nu/(u_* Pr)](\partial T/\partial y)|_w$ . And in this case, the results closest to experimental data are calculated using 2-liquid model.



**Fig. 3.** Dimensional excess tem profile

Turbulent stresses and temperature flux provide important information on turbulent flows. Therefore, Figures 4 and 5 show the profiles of turbulent stress and temperature flux in the same cross section  $\xi = 2.535$ . For these parameters, the best agreement with experimental measurements is also observed for the 2-liquid model.

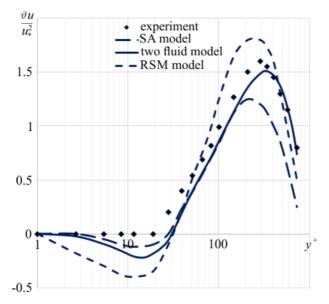


Fig. 4. Turbulent stress profile

It has been said above that, when applying RANS models for the numerical study of the problem under consideration, it is necessary to set the transition zone from the laminar regime to the turbulent regime on the basis of experimental data. Therefore, the ability of the 2-liquid model to predict this transition zone directly, without involving experimental data, is naturally of interest. Figure 5 shows the result of the 2-liquid model for a number of Nusselt  $Nu_x$  in dependence on the Rayleigh number Ra, which are related to the model parameters by relations.

$$\operatorname{Nu}_{x} = -\frac{x}{T_{w} - T_{0}} \frac{\partial T}{\partial y} \Big|_{w}, \quad \operatorname{Ra} = \operatorname{Pr}\operatorname{Re}_{w}^{2} x^{3},$$

where the Prandtl and Reynolds numbers are taken from [21]. The figure shows that, starting approximately from the  $Ra = 10^{9}$ , value, the turbulent regime begins and the Nusselt number begins to increase sharply compared to the value characteristic of the laminar regime. The results of the 2-liquid model describe well all flow regimes, including the

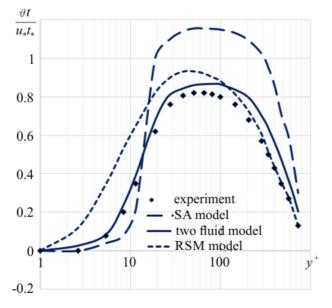


Fig. 5. Turbulent temperature flow profile

transition from the laminar to the turbulent regime.

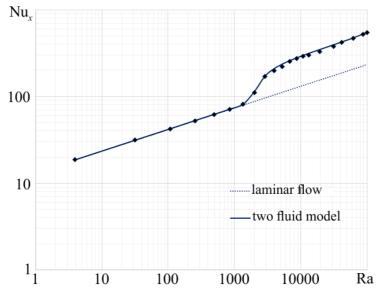


Fig. 6. Dependence of Nusselt number on Relay number according to the 2-liquid turbulence model

It should be noted that the turbulent thermal force plays an important role in modeling natural convection. Nusselt number calculations have shown that without taking this force into account, the transition from laminar to turbulent flow regime is not possible, i.e., the flow remains laminar even at large distances from the plate. The transition to the turbulent regime is not observed even with a strong increase in the initial perturbations, e.g., at  $\bar{u} = 0.1$ ,  $\bar{v} = 0$ . This circumstance testifies to the necessity to take into account the turbulent thermal force in calculations of turbulent natural convection, despite its smallness in comparison with other forces.

## 4. Conclusions

The numerical results obtained show that the 2-liquid model allows a more accurate description of natural convection than well known RANS models. It is proved that the thermal force that initiates the transition from laminar to turbulent regime without consideration of additional corrections must be taken into account when modeling natural convection. The 2-liquid model is simple for numerical realization and has good stability, so it can be recommended for studies of natural convection flows.

## References

- 1. *Lal S.A., Reji R.V.* Numerical Simulation of Natural Flow of Air Through a Room. International Journal of Green Energy. 2012. Vol. 9. P. 540–552. DOI: 10.1080/15435075.2011.622153.
- Chumakov Y., Levchenya A.M., Malah H. The vortex structure formation around a circular cylinder placed on a vertical heated plate. St. Petersburg Polytechnic State University Journal. Physics and Mathematics. 2018. Vol. 11, no. 1. P. 73–85. DOI: 10.18721/JPM.11108.
- 3. *Roul M.K.*, *Nayak R.C.* Experimental Investigation of Natural Convection Heat Transfer through Heated Vertical Tubes. International Journal of Engineering Research and Applications (IJERA). 2012. Vol. 2, no. 6. P. 1088–1096.
- 4. *Neilo R.V.* Experimental study of hydrodynamics and heat transfer in conditions of free convection. Technological audit and reserves of production. 2013. Vol. 6, 5(14). P. 169–171456.
- Ganesan P., Rani H.P. Transient natural convection along vertical cylinder with Heat and Mass transfer. Heat and Mass Transfer. 1998. Vol. 33. P. 449–455. DOI: 10.1007/s002310050214.
- 6. *Madera A.G.* Mathematical modeling of free convection of a vertical plate in the conjugate formulation. International Journal of Applied and Fundamental Research. 2015. No. 4–1. P. 25–28.

- 7. *Gorelikov A.V., Ryakhovsky A.V.* Numerical modeling of natural convection in a spherical layer. Bulletin of Tomsk State University. Mathematics and mechanics. 2008. No. 3. P. 77–84.
- 8. *Fetecau C., Vieru D., Azhar W.A.* Natural Convection Flow of Fractional Nanofluids Over an Isothermal Vertical Plate with Thermal Radiation. Applied Sciences. 2017. Vol. 7, no. 3. 247. DOI: 10.3390/app7030247.
- 9. *Martorell I., Herrero J., Grau F.X.* Natural convection from narrow horizontal plates at moderate Rayleigh numbers. International Journal of Heat and Mass Transfer. 2003. Vol. 46, no. 13. P. 2389–2402. DOI: 10.1016/S0017-9310(03)00010-3.
- Abdulateef J., Hassan A. Correlations for Nusselt Number in Free Convection from an Isothermal Inclined Square Plate by a Numerical Simulation. American Journal of Mechanics and Applications. 2015. P. 8–18. DOI: 10.11648/j.ajma.20150302. 11.
- 11. *Romanov S.V., Ziganshin A.M., Posokhin V.N.* Izvestiya VUZov. Construction. Izvestiya VUZov. Construction. 2007. Vol. 8. P. 111–115.
- 12. *Kholboev B.M., Navruzov D.P., Asrakulova D.S., Engalicheva N.R., Turemuratova A.A.* Comparison of the Results for Calculation of Vortex Currents After Sudden Expansion of the Pipe with Different Diameters. International Journal of Applied Mechanics and Engineering. 2022. Vol. 27, no. 2. P. 115–123. DOI: 10.2478/ijame-2022-0023.
- Shur M.L., Spalart P.R., Strelets M.K. Jet noise computation based on enhanced DES formulations accelerating the RANS-to-LES transition in free shear layers. International Journal of Aeroacoustics. 2016. Vol. 15, no. 6/7. P. 595–613. DOI: 10.1177/1475472x16659388.
- 14. *Bagaev D., Syralyova M.* Numerical simulation of free-convection flow near vertical heating surface. Transactions of the Krylov State Research Centre. 2018. Vol. 2, no. 384. P. 93–98. DOI: 10.24937/2542-2324-2018-2-384-93-98.
- 15. *Smirnov S., Smirnov E.* Experience of using semiempirical differential models of turbulence for calculation of liquid-metal convection in a bottom heated cylinder. 2022. DOI: 10.18721/JPM.15304.
- 16. *Ince N.Z., Launder B.E.* Computation of Turbulent Natural Convection in Closed Rectangular Cavities. Proc. 2nd U.K. National Conf. on Heat Transfer. Vol. 2. 1988. P. 1389–1400.
- 17. *Davidson L*. Second-order corrections of the *k*−ε model to account for non-isotropic effects due to buoyancy. International Journal of Heat and Mass Transfer. 1990. T. 33, № 12. C. 2599–2608. DOI: 10.1016/0017-9310(90)90195-z.
- Spalart P.R., Allmaras S.R. A One-Equation Turbulence Model for Aerodynamic Flows. La Recherche Aerospatial. 1994. No. 1. P. 5–21.
- Menter F. Zonal Two Equation k ω Turbulence Models For Aerodynamic Flows. 23rd Fluid Dynamics, Plasmadynamics, and Lasers Conference. American Institute of Aeronautics, Astronautics, 1993. 93-2906. DOI: 10.2514/6.1993-2906.
- 20. Wilcox D.C. Turbulence Modeling for CFD. DCW Industries. Inc. La Canada. California, 1993. 460 p.
- Bassina I.A., Lomakin S.A., Nikulin D.A., Strelets M.H., Shur M.L. Assessment of applicability of modern models of turbulence for calculation of natural convective flows and heat transfer. Thermophysics of High Temperatures. 1998. Vol. 36, no. 2. P. 230–238.
- 22. *Spalding D.B.* A turbulence model for buoyant and combusting flows. International Journal for Numerical Methods in Engineering. 1987. Vol. 24, no. 1. P. 1–23. DOI: 10.1002/nme.1620240102.
- 23. *Spalding D.B.* A turbulence model for buoyant and combusting flows. 4th Int. Conf. on Numerical methods in Thermal Problems. Swansea, 1984. Also, as Imperial College report CFD/86/4 (1984).
- 24. *Hoshoudy G.A.* Rayleigh-Taylor instability in quantum magnetized viscous plasma. Plasma Physics Reports. 2011. Vol. 37, no. 9. P. 775–784. DOI: 10.1134/s1063780x11080046.
- 25. *Markatos N.C., Kotsifaki C.A.* One-dimensional, two-fluid modelling of turbulent premixed flames. Applied Mathematical Modelling. 1994. Vol. 18. P. 646–657. DOI: 10.1016/0307-904x(94)90389-1.
- 26. *Shen Y.M.*, *Ng C.-O.*, *Chwang A.T.* A two-fluid model of turbulent two-phase flow for simulating turbulent stratified flows. Ocean Engineering. 2003. Vol. 30. P. 153–161. DOI: 10.1016/s0029-8018(02)00020-3.
- 27. *Starchenko V., Nuterman R.B., Danilkin E.A.* CHislennoe modelirovanie turbulentnyh techenij i perenosa primesi v ulichnyh kan'onah. Tomsk: Tomskij gosudarstvennyj universitet, 2015. 252 c.
- 28. *Malikov Z*. Mathematical model of turbulence based on the dynamics of two fluids. Applied Mathematical Modelling. 2020. Vol. 82. P. 409–436. DOI: 10.1016/j.apm.2020.01.047.

- 29. *Malikov Z.M.* Mathematical model of turbulent heat transfer based on the dynamics of two fluids. Applied Mathematical Modelling. 2021. Vol. 91. P. 409–436. DOI: 10.1016/j.apm.2020.09.029.
- 30. *Tsuji T., Nagano Y.* Characteristics of a turbulent natural convection boundary layer along a vertical flat plate. International Journal of Heat and Mass Transfer. 1988. P. 1723–1734.
- 31. *Tsuji T., Nagano Y.* Turbulence measurements in a natural convection boundary layer along a vertical flat plate. International Journal of Heat and Mass Transfer. 1988. Vol. 31, no. 10. P. 2101–2111. DOI: 10.1016/0017-9310(88)90120-2.
- 32. Von Mises R. Bernerkungen zur Hydrodynamik. Zeitschrift für Angewandte Mathematik und Mechanik. 1927. Vol. 7. P. 425.
- 33. *Malikov Z., Navruzov D., Djumayev X.* Models results Comparison of different approaches to turbulence for flow past a heated flat plate. E3S Web of Conferences. 2021. Vol. 264. 01008. DOI: 10.1051/e3sconf/202126401008.

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