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EVALUATION OF THE INFLUENCE OF A DIMENSIONLESS THRESHOLD PRESSURE GRADIENT ON DEVIATION OF FILTRATION FROM CLASSICAL FILTRATION LAW

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The depletion of hydrocarbon reserves in reservoirs with high and medium permeability (rocks containing pores, cavities or fracture systems) leads to an increase in the proportion of low permeability reservoirs. In such reservoir rocks, nonlinear filtration effects are observed: at low pressure gradients, filtration does not obey the classical filtration law (Darcy's law). One of the methods to describe the nonlinear filtration effects is to apply the filtration law with a threshold pressure gradient (TPG). The influence of this parameter on the deviation of filtration from Darcy's law has not been previously analyzed. The aim of this work is to evaluate the influence of TPG on the filtration process. In the article, the equations of piezoconductivity are nondimensionalized and solved according to the classical filtration law and the filtration law with TPG under conditions of plane radial flow and maintaining a constant well pressure. In the framework of the classical filtration law, the piezoconductivity equation is solved by applying a self-simulated variable. Using the filtration law with TPG, the conductivity equation solution is solved by the method of integral relations. As a result, the dependences of the dimensionless production rate and dimensionless time are found for both filtration laws from the solved piezoconductivity equations. To confirm the identity of the obtained solutions, a comparative analysis was carried out at a zero threshold pressure gradient. A comparison was made of the dependences of the production decline curves on time when filtering according to the classical filtration law and the filtration law with TPG at its various values. The dependence of the form of the production decline curves on the value of the limiting pressure gradient is analyzed. It was found that with increasing dimensionless TPG, the influence of nonlinear effects on filtration increases: nonlinear filtration effects begin to manifest themselves much earlier, and the production decline curves during filtration according to the law with TPG deviate more strongly from the production decline curve under the classical filtration law.

Key words: continuum mechanics, method of integral relations, unconventional oil reserves, low permeability reservoir, threshold pressure gradient (TPG), nonlinear filtration, production decline curve

1. Introduction

Currently, due to the depletion of traditional oil reserves, there is an increasing need to develop unconventional oil reserves [1]. These reserves include reservoirs with low permeability, that is a parameter that characterizes the ability of a porous medium to pass liquid through itself, or to filter [2], in which, as studies of filtration in the core show, at low pressure gradients filtration deviations from the classical law take place [3, 4]. Nonlinear effects in low-permeable reservoirs are also observed in filtration experiments with injection of water at residual oil saturation and 100% water saturation [5], oil at residual water saturation [5]; two-phase filtration of water and oil in different ratios [5]; suspensions that are non-Newtonian liquids [6]. At the same time, under the conditions of high liquid filtration rates, the binomial Forchheimer law is used instead of Darcy's law [7].

For a mathematical description of nonlinear effects of filtration, in particular, the threshold pressure gradient (TPG) is used. In some sources, it is defined as the value of the pressure gradient at the point of intersection with the abscissa axis of filtration rate dependence from the pressure gradient (pressure gradients). At pressure gradients less than TPG, the filtration rate is considered to be zero [8]. In other sources, the value of the pressure gradient is taken as TPG, starting from which the tangent of the slope angle of filtration rate dependence on pressure gradient becomes constant, and the value of the pressure gradient at the intersection point of the filtration rate curve, as its function, with the abscissa axis is called the initial pressure gradient [5]. Sometimes the filtration law with TPG is called quasi-linear.

There are also other types of dependence between the filtration rate and the pressure gradient that describe the nonlinear effects of filtration: the power law [9], empirical dependencies [5], piecewise linear functions [10], representation of permeability as a function of the pressure gradient determined on the basis of experimental or field data [8].

Numerical models can be applied to the calculations of flows in low-permeable reservoirs taking into account filtration deviations from the classical law using various nonlinear laws. These models allow forecasts that are more accurate compared to models that use the classical filtration law [4, 8]. Thus, based on numerical experiments in [10], it is concluded that it is necessary to take into account nonlinear effects of filtration in

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ultra-low-permeability reservoirs. In [11], according to the results of numerical simulation under variable operating modes of the well, it was found that nonlinear filtration effects affect the flow rate of the well. It was established that the higher the TPG value, the more the flow rate decreases as compared to the flow rate under the classical filtration law. However, a detailed analysis of the relationship between the level of the threshold pressure gradient and the dynamics of the oil production decline has not been carried out, which determines the relevance of this work.

The aim of the work is to evaluate the influence of the TPG value on the intensity of the manifestation of nonlinear filtration effects in a reservoir with a vertical well with the constant bottom-hole pressure by comparing the production decline curve during filtration according to the classical law [12] and according to the law with TPG at its various values. An approximate analytical solution for the oil flow rate with refined initial conditions corresponding to the radial formulation of the problem is obtained (the concept of the radial flow regime in the reservoir is fundamental to the theory of hydrodynamic studies), which is compared with the solution based on the classical filtration law in the absence of TPG. For the first time, characteristic dimensionless moments of time are introduced, at which the dynamics of the oil flow rate, calculated according to the law with TPG, begins to deviate from the dynamics of the flow rate according to the classical filtration law. The calculation will make it possible to clarify the forecast of oil production at a real field.

2. Statement of the problem and introduction of dimensionless parameters

To achieve the stated aim, it is necessary to find dimensionless flow rates during filtration according to the classical filtration law and the filtration law with TPG at a constant pressure in a vertical well, which is developed by traditional methods, and then compare the obtained flow rate dependences on time with each other.

The initial condition for the problem under consideration is as follows: before the well is launched, the pressure p throughout the entire reservoir is the same:

$$p(r,0) = p_0, \tag{1}$$

where p_0 is reservoir pressure, r is radial coordinate.

After setting the initial conditions (1), it is necessary to set the boundary conditions. The bottom-hole pressure at all times $t > 0$ is maintained equal to:

$$p(r_w, t > 0) = p_w, \tag{2}$$

where r_w is the well radius, p_w is the bottom-hole pressure. At an infinite distance from the well, the pressure at all times t is equal to reservoir pressure:

$$p(r \to \infty, t) = p_0. \tag{3}$$

The piezoconductivity equation is obtained from the mass conservation law for slightly compressible fluid and filtration law. After reduction of the piezoconductivity equation to a dimensionless form and its solution the dependence of production rate on time is obtained.

The mass conservation law for slightly compressible fluid is as follows:

$$c_t m \frac{\partial p}{\partial t} + \mathrm{div}\mathbf{u} = 0, \tag{4}$$

where c_t is coefficient of total compressibility of the reservoir system, m is porosity, which is equal to the ratio of pore volume to the total reservoir volume, $\mathbf{u}$ is the filtration velocity vector.

The filtration law is written as [12]:

$$\nabla p = -\Pi \Phi\left(\frac{u}{\lambda}\right)\frac{\mathbf{u}}{u}, \tag{5}$$

where λ is characteristic value of the filtration rate, Π is characteristic value of the pressure gradient, u is filtration velocity, $\Phi(u/\lambda)$ is function, which establishes the dependence between the pressure gradient and filtration velocity. The filtration law can also be written is another form if the filtration rate is expressed in terms of pressure gradient [9]:

$$\mathbf{u} = -\lambda\psi\left(\frac{|\nabla p|}{\Pi}\right)\frac{\nabla p}{|\nabla p|}; \tag{6}$$

where $\psi(|\nabla p|/\Pi)$ is function, which establishes the dependence between filtration velocity and pressure gradient.

The piezoconductivity equation is obtained as a result of the substitution of the filtration velocity (6) to the mass conservation law (4). At the same time, it is assumed that the flow is radial one-dimensional:

$$\frac{\partial p}{\partial t} = \frac{\lambda}{c_t m}\frac{1}{r}\frac{\partial}{\partial r}\left[r\psi\left(\frac{1}{\Pi}\frac{\partial p}{\partial r}\right)\right]. \tag{7}$$

Values are reduced to dimensionless form using the following relations:

$$P = \frac{p - p_w}{p_0 - p_w}, \tag{8}$$

$$R = \frac{r}{r_w}, \tag{9}$$

$$T = \frac{\kappa t}{r_w^2}, \tag{10}$$

$$Q = \frac{\mu q}{2\pi k h(p_0 - p_w)}. \tag{11}$$

Here P, R, T, Q are dimensionless pressure, radial coordinate, time and production rate; κ is piezoconductivity coefficient; q is dimensional production rate (flow rate of produced fluid); μ is fluid viscosity; k is permeability; h is reservoir thickness. The piezoconductivity coefficient κ is determined by the formula:

$$\kappa = \frac{k}{c_t m \mu}. \tag{12}$$

Boundary conditions (2) and (3) can be written using equations (8) and (9):

$$P(R=1) = \frac{p_w - p_w}{p_0 - p_w} = 0, \tag{13}$$

$$P(R\to\infty) = \frac{p_0 - p_w}{p_0 - p_w} = 1. \tag{14}$$

3. Solution of the problem according to the classical filtration law

Solution of the problem according to the classical filtration law is obtained using self-similar variable [13]. The following relations are valid for the classical filtration law: $\Pi = 1$, $\lambda = k/\mu$, $\psi = \partial p/\partial r$. Then the equation (7) can be written as:

$$\frac{\partial p}{\partial r}=\frac{k}{c_t m\mu}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}\right]. \tag{15}$$

After calculating the derivatives in (15) and substituting the piezoconductivity coefficient (12), the equation (15) takes the form:

$$\frac{\partial p}{\partial t}=\kappa\left(\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}\right). \tag{16}$$

The equation (16) is reduced to dimensionless form as a result of replacing variables according to formulas (8), (9):

$$\frac{\partial P}{\partial T}=\frac{\partial^2 P}{\partial R^2}+\frac{1}{R}\frac{\partial P}{\partial R}. \tag{17}$$

The self-similar variable is introduced as follows:

$$\xi=\frac{R}{2\sqrt{T}}. \tag{18}$$

Determination of the pressure derivatives by the self-similar variable (18), their substitution in (17) lead to the expression:

$$\frac{d^2 P}{d\xi^2}+\left(\frac{1}{\xi}+2\xi\right)\frac{dP}{d\xi}=0. \tag{19}$$

As a result of the reverse substitution of the value of the self-similar variable (18) and taking into account the boundary conditions (13) and (14), the solution of the differential equation (19) takes the form:

$$P=1-\mathrm{Ei}\left(-R^2/(4T)\right)/\mathrm{Ei}\left(-1/(4T)\right), \tag{20}$$

where $\mathrm{Ei}\left(-1/(4T)\right)$ is exponential integral of the argument $-1/(4T)$.

Production rate is calculated using the formula:

$$q=\frac{2\pi kh}{\mu}r_w\left(\frac{\partial p}{\partial r}\right)_{r=r_w}. \tag{21}$$

According to equations (8), (9) and (11), production rate (21) can be written as:

$$Q=\left(\frac{\partial P}{\partial R}\right)_{R=1}. \tag{22}$$

Substituting (20) into (22) allows us to obtain the dependence of the flow rate on time according to the classical filtration law:

$$Q=-\left(2/\mathrm{Ei}\left(-1/(4T)\right)\right)\exp^{-1/(4T)}. \tag{23}$$

4. Solution of the problem for filtration with threshold pressure gradient

In the case of filtration with a threshold pressure gradient, the solution is found using the method of integral relations [12]. Since in equation (7) the pressure is contained only under the sign of the derivative, it is possible to write a pressure perturbation instead:

$$p_d = p - p_0. \tag{24}$$

The disturbance zone will further be understood as the reservoir area where the pressure differs significantly from the reservoir. The boundary of this region is removed from the well and is called the boundary of the disturbance zone.

The distribution of filtration velocities in disturbance zone is equal to:

$$u(r,t) = \frac{q_g(t)}{2\pi r h}\left[1 - \frac{r}{l(t)}\right], \qquad r \le l(t), \tag{25}$$

where q_g is production flow rate for filtration with TPG, $l(t)$ is the coordinate of the boundary of the disturbance zone as a function of time, which is found using material balance method [12]:

$$\frac{d}{dt}\int_0^l 2\pi r m h c_t p_d(r,t)\,dr = -q_g(t). \tag{26}$$

In the filtration law (5), the pressure under the gradient sign can be replaced by the pressure perturbation (24), then the expression can be written as:

$$p_d(r,t) = -\int_r^{l(t)} \Pi\Phi\left(\frac{|u(r,t)|}{\lambda}\right)dr. \tag{27}$$

The following relations are valid for the filtration law with TPG: $\Pi = G$, $\lambda = kG/\mu$, $\Pi(u/\lambda) = u/\lambda + 1$, where G is the threshold pressure gradient. After substituting them in (27), a formula for finding the pressure perturbation is obtained:

$$p_d(r,t) = -\int_r^{l(t)} G\left(\frac{\mu}{kG}u + 1\right)dr, \tag{28}$$

and the dimensionless TPG and the position of the boundary of the disturbance zone are found by the formulas:

$$\Gamma = G\frac{r_w}{p_0 - p_w}, \tag{29}$$

$$L = \frac{l}{r_w}. \tag{30}$$

Substitution in (28) of the filtration rate values from (25), taking into account the formulas (8), (9), (29), (30) and the calculation of the integral leads to the expression:

$$P = 1 - Q_g\left[\ln\left(\frac{L}{R}\right) + \left(1 - \frac{R}{L}\right)\left(\frac{\Gamma}{Q_g}L - 1\right)\right], \tag{31}$$

where Q_g is the dimensionless production rate for filtration with TPG.

Next, it is necessary to substitute the boundary condition (13) in (31) and express the value of the production rate:

$$Q_g = \frac{1 - \Gamma(L-1)}{\ln(L) + \dfrac{1}{L} - 1}, \qquad L \ge 1. \tag{32}$$

The inequality in (32) follows from the condition for (25) in this case.

The reduction of material balance equation (26) to the dimensionless form can be done using the formulas (8)–(10), (22):

$$\frac{d}{dT}\int_0^L R\left(P-1\right)dR=-Q_g. \tag{33}$$

Taking into account the pressure (31) in equation (33) and the transformations give an equation for determining the position of the boundary of the disturbance zone:

$$\frac{d}{dT}\left(Q_g L^2+2\Gamma L^3\right)=12Q_g. \tag{34}$$

The boundary of the disturbance zone should go beyond the outer boundary of the well, but not reach the zone where the pressure becomes equal to the reservoir pressure. Then in (34), taking into account (29) and (30), the condition is set on the coordinate of the dimensionless boundary of the disturbance zone:

$$1 << L << \Gamma^{-1}, \tag{35}$$

where the left inequality corresponds to the removal of the boundary of the disturbance zone from the outer boundary of the well, and the right one corresponds to its inability to reach the zone where the pressure is equal to the reservoir, since in this case there is a pressure drop p_0-p_w (see (29)). Under condition (35), the equation (34) can be simplified by using the definition of the derivative of a complex function. The result is a differential equation:

$$2LdL\approx 12dT. \tag{36}$$

The solution of differential equation (36) can be obtained with the initial condition $L\left(T=0\right)=1$, which corresponds to the dimensionless coordinate of the disturbance zone equal to well radius (according to (30)):

$$L\approx\sqrt{12T}+1. \tag{37}$$

It should be noted that it is not possible to implement a numerical solution of the differential equation (34) due to the initial condition under which the denominator of the expression (32) necessary for calculating the oil flow rate turns to zero. Therefore, the article proposes an approximate analytical method for calculating the flow rate of oil during filtration with TPG.

The dependence of the flow rate on time for filtration law with TPG can be obtained by substituting the coordinate of the dimensionless boundary of the disturbance zone from (37) into the equation for finding the flow rate (32) and taking into account the condition (35):

$$Q_g=\frac{1-\Gamma\left(\sqrt{12T}\right)}{\ln\left(\sqrt{12T}+1\right)+\dfrac{1}{\sqrt{12T}+1}-1},\qquad 1<<\sqrt{12T}+1<<\Gamma^{-1}. \tag{38}$$

5. Results and discussion

Next, it is necessary to compare the curves of the flow rate drop during filtration according to the classical law and according to the filtration law with TPG at $\Gamma=0$. Since there is no TPG in this case, and filtration occurs according to the classical law, the solutions according to the two laws must coincide with each other.

For calculations, the classical filtration law is used in the form of (23) with an exponential integral found with an accuracy of 1% [13].

The equation for production flow rate according to the filtration law with TPG at $\Gamma=0$ becomes the following:

$$Q_g = \frac{1}{\ln\left(\sqrt{12T}+1\right)+\frac{1}{\sqrt{12T}+1}-1}. \qquad (39)$$

Figure 1 shows the dependences of the flow rate on time on a semi-logarithmic scale, calculated by formulas (23) and (39). The analysis of the curves shows a satisfactory correspondence of solutions for the classical filtration law and the filtration law with TPG for the entire calculated time range. From a comparison of the flow rate graphs for the classical filtration law and the filtration law with TPG, it can be concluded that the flow rate values calculated by formula (38) are correct, and they can be used to estimate the flow rates for filtration law with TPG.

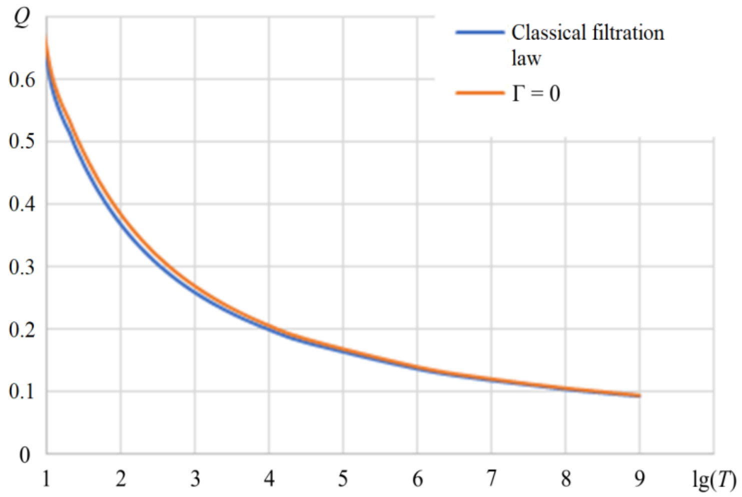

Fig. 1. For comparison of the production decline curves for classical filtration law and the filtration law with threshold pressure gradient at $\Gamma = 0$.

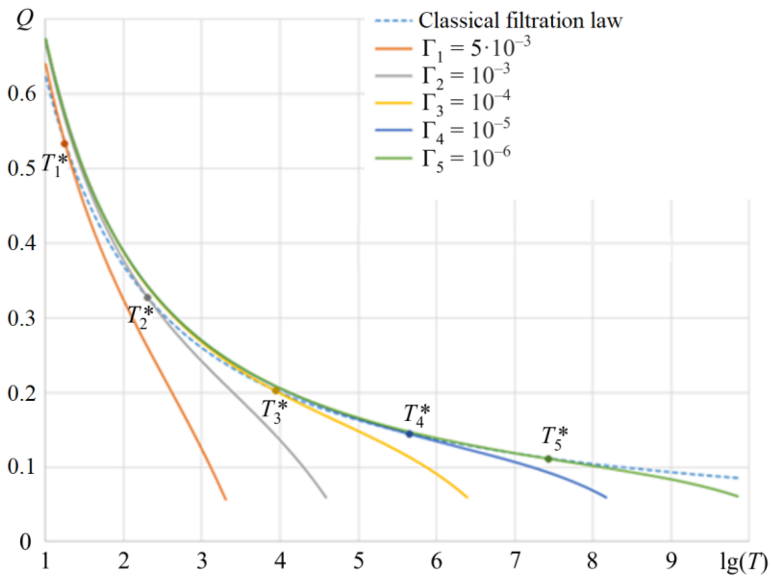

Fig. 2. Comparison of the production decline curves for the classical filtration law and the filtration law with threshold pressure gradient for different values of Γ.

Solutions were obtained for the filtration law with TPG at various values Γ (Fig. 2). Here, for comparison, the dotted line shows the production decline curve for the classical filtration law (23). The dots mark its intersections with the curves of the production decline curve according to the law with TPG, which are calculated according to the formula (38) for different values of the threshold pressure gradient Γ. The dots of curves intersections correspond to the dimensionless time T^*, which means the moment of the beginning of the filtration deviation from the classical law. From the time T^* production flow rate for filtration law with TPG declines faster than the production flow rate for the classical filtration law. At time $T < T^*$ filtration with TPG do not differ from filtration with the classical law. It can also be seen from the Figure 2 that when the value of threshold pressure gradient Γ decreases, the time until the beginning of the filtration deviation from the classical law (T^*) increases, that is, in reservoirs with small value of Γ nonlinear filtration effects appear later, and the values of production flow rate correspond to production flow rates in the case of the classical filtration law longer. It can be seen in figure 2, that for small values of Γ production decline curves deviate from a similar curve for classical filtration law ($T > T^*$) slower than at large values of Γ.

Table 1 shows the times of the beginning of the filtration deviation from the classical law T^* for different values of TPG Γ. The data from Table 1 can be converted to dimensional values using formulas (10), (12) and (29) at $r_w = 0{,}1\,\text{m}$, $k = 0{,}25\,\text{mD}$, $p_0 - p_w = 10\,\text{MPa}$, $c_t = 1{,}5 \cdot 10^{-3}\,\text{MPa/m}$, $\mu = 3\,\text{mPa·s}$, $m = 0{,}1$. The obtained values are given in Table 2.

Table 1. Dimensionless times of the beginning of filtration deviation from classical law T^*, corresponding to values Γ (dimensionless TPG).

Γ	$5\cdot10^{-3}$	10^{-3}	$5\cdot10^{-4}$	10^{-4}	$5\cdot10^{-5}$	10^{-5}	$5\cdot10^{-6}$	10^{-6}
T^*	17.7	199	606	8702	$2.82\cdot10^{4}$	$4.55\cdot10^{5}$	$1.54\cdot10^{6}$	$2.72\cdot10^{7}$

Table 2. Dimensional values of threshold pressure gradient and corresponding to them values of time of the beginning of filtration deviation from classical law t^* for set parameters.

G, Pa/m	$5\cdot10^{5}$	10^{5}	$5\cdot10^{4}$	10^{4}	$5\cdot10^{5}$	10^{5}	$5\cdot10^{2}$	10^{2}
t^*	5 minutes	1 hour	3 hours	2 days	6 days	3 months	11 months	15.5 years

Table 2 shows that at large values of TPG the time of beginning of the filtration deviation from the classical law t^* with the considered parameters appears quickly and is on the order of several minutes and hours, but at low values of TPG, the time t^* can reach several years. It is possible that for small values of TPG, nonlinear filtration effects will not have time to significantly reduce the production flow rate during its development, or nonlinear filtration effects will not have time to appear, if the time t^* will exceed the development time.

It can be established from the obtained graphs of the flow rate dependence on time in the case of filtration law with TPG and their comparison with a similar graph of dependence for classical filtration law (Fig. 2) that the threshold pressure gradient Γ characterizes the degree of influence of nonlinear effects on the filtration in the reservoir. The more Γ, the earlier and more noticeably the production decline curve for filtration law with TPG deviates from the production decline curve for the classical filtration law. Based on the TPG equation (29), it can be concluded that the role of nonlinear filtration effects in the production rate decline can be reduced by reducing the bottom-hole pressure p_w of the well drilled into the pay zone of the reservoir.

6. Conclusion

Thus, in the work, the dependences of the production flow rate on time at a constant pressure in a vertical well developed by traditional methods are obtained in the cases of the classical filtration law and the filtration law with TPG. It is established from the comparison of the production decline curves for filtration law with TPG and various values Γ with the production decline curve for classical filtration law that Γ characterizes the degree of the filtration deviation from the classical law: the greater the value of TPG, the sooner the deviation begins. Also, with an increase in the value Γ, there is a sharper divergence of the production decline curves after reaching the time T^* for the filtration law with TPG.

The study shows that in order to reduce the influence of nonlinear effects on filtration, it is necessary to reduce the level of the threshold pressure gradient Γ, and for this reason it is necessary to set the smaller values of the bottom-hole pressure p_w. It is possible that at large reservoir depressions (the difference between the reservoir pressure in the area of the well and its bottom-hole pressure, causing the movement of gas from the reservoir to the bottom-hole of the well), the value of the dimensional time of the beginning of the filtration deviation from the classical law t^* may exceed the operating time of the well and, consequently, the nonlinear filtration effects will not have time to affect the flow rate during the development of the well.

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