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# MODELING OF ELASTOPLASTIC DEFORMATION OF STEEL 45 ALONG ARCHIMEDES SPIRAL TYPE TRAJECTORIES

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This paper addresses the mathematical modeling of complex elastoplastic deformation of steel 45 along the plane trajectory in the Ilyushin's deviatoric space, which consists of sections of both constant and variable curvature (Archimedes spiral). The constitutive equations of the proposed mathematical model are based on the Ilyushin's vector representation of strain and stress. An approximate model of the theory of elastoplastic processes is used in mathematical modeling for plane trajectories with approximations of process functionals, which depend on the initial value of the curvature, rather than on the current curvature of the deformation trajectory. The constitutive equations of the mathematical model are reduced to the Cauchy problem, a numerical solution to which is obtained using the fourth order Runge–Kutta method. The validity of the mathematical model for this class of curvilinear strain trajectories was verified by comparing the calculation results with the experimental data obtained on the automated test machine SN-EVM in the mechanical testing laboratory of the Tver State Technical University. The experiment was carried out on thin-walled cylindrical specimens of steel 45 under complex loading (combined tension-compression and torsion). The calculation results and experimental data characterizing the scalar and vector properties of the material are presented graphically. It has been established that the proposed approximate mathematical model is able to capture (both qualitatively and quantitatively) the main effects of complex plastic deformation for the considered class of strain trajectories in the areas of small and medium curvature. More accurate calculation results in the approximations of the plasticity functionals can be obtained by taking into account all complex loading parameters, including the current curvature of the strain trajectory, especially for strain trajectories with large curvature.

***Key words**:* plasticity, complex loading, Ilyushin's theory of elastoplastic processes, mathematical model, strain trajectory, Archimedes spiral, vector and scalar material properties

## 1. Introduction

Experimental research and studying the revealed patterns of elastic-plastic deformation of structural materials is of great importance for the development of the theory of plasticity. On the basis of experimental data, new mathematical theories are formulated, and their physical validity and limits of applicability are evaluated. The results of experimental work under complex loading of materials and variants of mathematical theories of plasticity are partially presented in publications [1–18].

Considerable amount of experimental data has been obtained upon deformation of materials along plane rectilinear trajectories and curved trajectories of constant curvature [3, 6, 7, 12, 15–18]. For a more complete verification of the constitutive equations of the theory of plasticity, it is necessary to set up experiments with the widest possible range of changes in the curvature of the trajectory within one test. There have been significantly fewer such experiments. Here, of particular interest are curved deformation trajectories of the Archimedes spiral type [3, 6, 14], in which various curvatures, from small to large, are observed in one experiment.

In this article, we consider a mathematical model of the theory of elastic-plastic processes for deformation trajectories with sections of constant and variable curvature. The estimation of its reliability is carried out by comparing the results of calculations with experimental data [14] obtained on the computational test machine SN-EVM for this class of deformation trajectories with the sections described analytically by the Archimedes spiral. Model and experimental data are presented in the vector representation of strains and stresses according to A. A. Ilyushin [1–4].

Previously, the variants of the mathematical model used were used for numerical description of processes within the framework of the theory of elastic-plastic deformation processes of two-link and multi-link polyline rectilinear trajectories [15, 16], as well as trajectories containing curved sections of constant curvature [17, 18].

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## 2. Constitutive equations

We introduce the orthonormal basis $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, where $\delta_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j = \begin{cases} 0 & (i \neq j) \\ 1 & (i = j) \end{cases}$ — the Kronecker symbol $(i, j = 1, 2, 3)$. In this basis, the Cauchy stress tensor $\mathbf{T}_\sigma$ and the small strain tensor $\mathbf{T}_\varepsilon$, which characterize the stress-strain state of a body at a point with coordinates $x_n$ $(n = 1, 2, 3)$, it is customary to decompose into hydrostatic tensors and deviator tensors. In the component form, the latter are represented as follows [1–3]:

$$\sigma_{ij} = \sigma_0 \delta_{ij} + \sigma S^*_{ij}, \qquad \varepsilon_{ij} = \varepsilon_0 \delta_{ij} + Э Э^*_{ij} \qquad (i, j = 1, 2, 3).$$

Here, respectively: $\sigma_0 = \frac{1}{3}\sigma_{ij}\delta_{ij}$, $\varepsilon_0 = \frac{1}{3}\varepsilon_{ij}\delta_{ij}$, $\sigma = \sqrt{S_{ij}S_{ij}}$, $Э = \sqrt{Э_{ij}Э_{ij}}$ — modules of hydrostatic tensors (mean stress and strain) and stress and strain deviator tensors; $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0$, $Э_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_0$, $S^*_{ij} = \frac{S_{ij}}{\sigma}$, $Э^*_{ij} = \frac{Э_{ij}}{Э}$ — components of deviator tensors and direction tensors of stresses and strains $(i, j = 1, 2, 3)$.

In the case of simple, i.e. proportional loading, the components of the stress and strain direction tensors coincide: $S^*_{ij} = Э^*_{ij}$. Then, taking into account the elastic volume deformation, the constitutive equations (according to the theory of small elastic-plastic deformations) will take the form [1–3]:

$$\sigma_0 = 3K\varepsilon_0, \qquad S_{ij} = \frac{\sigma}{Э} Э_{ij} = 2G_p Э_{ij} \qquad (i, j = 1, 2, 3),$$

where $K$ — Bulk modulus, $G_p$ — plastic shear modulus, $\sigma = \Phi(Э)$ — a universal Roche–Eichinger function under simple loading, which determines the scalar properties of materials for an arbitrary stress-strain state. When implementing complex loading, the components of the stress and strain direction tensors do not coincide: $S^*_{ij} \neq Э^*_{ij}$. This means that taking into account only the scalar properties of materials in the determining ratios is insufficient. However, the tensor presentation of the theory of plasticity cannot geometrically visualize the vector properties of materials in three-dimensional physical space [3].

A. Ilyushin proposed [1, 2] a vector (geometric) representation of the process of deformation or loading, where in a linear combined Euclidean space $E_6$ with an orthonormal fixed basis $\{\mathbf{i}_k\}$ $(k = 0, 1, 2, \ldots, 5)$ tensors with components $\sigma_{ij}$ and $\varepsilon_{ij}$ the stress and strain vectors are placed in one-to-one linear correspondence:

$$\mathbf{S} = S_0 \mathbf{i}_0 + \boldsymbol{\sigma}, \qquad \boldsymbol{\varepsilon} = Э_0 \mathbf{i}_0 + \mathbf{Э},$$

where $\boldsymbol{\sigma} = S_k \mathbf{i}_k$, $\mathbf{Э} = Э_k \mathbf{i}_k$ $(k = 1, 2, \ldots, 5)$ — vectors of stresses and strains of the shape change in the 5 dimensional deviator subspace $E_5$. The validity of such a representation is discussed in detail in [5]. The coordinates of vectors are related to the components of tensors and deviators by one-to-one relations [1–4]:

$$S_0 = \sqrt{3}\sigma_0, \quad S_1 = \sqrt{\frac{3}{2}}S_{11}, \quad S_2 = \frac{S_{22} - S_{33}}{\sqrt{2}}, \quad S_3 = \sqrt{2}S_{12}, \quad S_4 = \sqrt{2}S_{23}, \quad S_5 = \sqrt{2}S_{13},$$
$$Э_0 = \sqrt{3}\varepsilon_0, \quad Э_1 = \sqrt{\frac{3}{2}}Э_{11}, \quad Э_2 = \frac{Э_{22} - Э_{33}}{\sqrt{2}}, \quad Э_3 = \sqrt{2}Э_{12}, \quad Э_4 = \sqrt{2}Э_{23}, \quad Э_5 = \sqrt{2}Э_{13}.$$

Modules of vectors in a 5-dimensional subspace $E_5$ are equal to the modules of the stress and strain deviator tensors, respectively:

$$|\boldsymbol{\sigma}| = \sigma = \sqrt{S_k S_k} = \sqrt{S_{ij} S_{ij}}\,, \qquad |\mathbf{Э}| = Э = \sqrt{Э_k Э_k} = \sqrt{Э_{ij} Э_{ij}} \quad (k = 1, 2, \ldots, 5)\,,\ (i, j = 1, 2, 3)\,.$$

In the theory of elastic-plastic processes of A. A. Ilyushin [1–4], the history of changes in stresses and strains over time is represented as an image of deformation or loading — a trajectory, to each point of which variable characteristics of the process (vectors of stresses, strains and their increments) and scalar parameters (temperature, average stress and strain, and others) are attributed [1–4]. The general constitutive equations of the theory of plasticity [3, 4] reflect the relationship between the vectors of stresses $\boldsymbol{\sigma}$ and strains of the shape $\mathbf{Э}$ change taking into account the scalar and vector properties of materials. Scalar properties define the relationship between the invariants of stress and strain deviator tensors, and vector properties describe the misalignment of stress and strain deviator tensors and their increments. For the case of plane strain trajectories with analytical curved sections, the constitutive equations have the form [3, 4]:

$$\frac{d\boldsymbol{\sigma}}{ds} = M_1 \frac{d\mathbf{Э}}{ds} + \left( \frac{d\sigma}{ds} - M_1 \cos\vartheta_1 \right) \frac{\boldsymbol{\sigma}}{\sigma}, \qquad \frac{d\vartheta_1}{ds} + \kappa_1 = -\frac{M_1}{\sigma} \sin\vartheta_1. \tag{1}$$

Here: $s$ — arc length of the strain trajectory; $\vartheta_1 = \vartheta_1(s, \kappa_1, \vartheta_1^0)$ — the angle of delay, which is a functional of the vector properties of the material, which determines the direction of the vector at each point of the strain trajectory and the influence of the vector properties of the material on the deformation process; $\vartheta_1^0$ — the angle of the trajectory break at the starting point of its analytical section; $\kappa_1$ — trajectory curvature; $\sigma = \sigma(s, \kappa_1, \vartheta_1^0)$ — functional of scalar properties of a material; $M_1$, $\dfrac{d\sigma}{ds}$ — functionals of the deformation process, depending on the parameters of complex loading $s, \kappa_1, \vartheta_1^0$.

## 3. Mathematical model of elastic-plastic processes

The basic equations of the mathematical model in the framework of the theory of elastic-plastic deformation processes include the constitutive equations (1) and approximations of the functional processes [3]. For plane curved trajectories with sections of variable curvature such as the Archimedes spiral, the following approximations of the functionals are used in the model:

$$\begin{gathered} \sigma(s) = \Phi\left(s, \vartheta_1^0, \kappa_0\right) = \Phi\left(s\right) + A f_0^{\,p}\, \Omega - B_1 \kappa_0 e^{-\alpha_1 (s_{\max} - s)} - 2G_* \Delta s, \\ M_1 = 2G_p + \left(2G - 2G_p^0\right) f^{\,q}. \end{gathered} \tag{2}$$

In expressions (2), the notation is accepted: $\Phi\left(s\right)$ — universal Odquist-Ilyushin function for processes close to simple loading without taking into account their history; $\Delta s = s - s_K^{\text{т}}$ — the increment of the arc of the strain trajectory, where $s_K^{\text{т}}$ — the length of the arc at the point of the trajectory break or change in its curvature (at the junction of the analytical sections); $s_{\max}$ — arc length for the end point of the strain trajectory on the Archimedes spiral section; $\kappa_0$ — the value of the curvature of the strain trajectory at the starting point of the Archimedes spiral section; $G$ — shear modulus; $G_p^0$ — calculation of the plastic shear modulus $G_p$ at the break point of the trajectory; $\Omega$ — the complex loading function [3], which describes the scalar "dive" of stresses at the trajectory break and the generalized Bauschinger effect at the resulting complex unloading and subsequent secondary plastic deformation,

$$\Omega = -\left[\gamma \Delta s\, e^{-\gamma \Delta s} + b\left(1 - e^{-\gamma \Delta s}\right)\right]; \tag{3}$$

$f = f(\vartheta_1) = \dfrac{1-\cos\vartheta_1}{2}$; $f$ — the complex loading function [3], which takes into account the orientation of the stress vector during deformation process and its value at the point of the strain trajectory break, $f_0 = f(\vartheta_1^0) = \dfrac{1-\cos\vartheta_1^0}{2}$; $A, B_1, \alpha_1, b, \gamma, p, q$ — numerical parameters of approximations.

To approximate the universal Odquist–Ilyushin hardening function under simple (proportional) loading, the expressions [3]

$$\sigma = \Phi(s) = \begin{cases} \dfrac{2G}{\alpha}\left(1 - e^{-\alpha s}\right), & 0 \le s \le s^{\text{т}}, \\ \sigma^{\text{т}} + 2G_*(s - s^{\text{т}}) + \sigma_*\left(1 - e^{-\beta(s - s^{\text{т}})}\right), & s > s^{\text{т}}. \end{cases} \tag{4}$$

Here: $\sigma^{\text{т}} = \sqrt{2/3}\,\sigma_{\text{т}}$; $\sigma_{\text{т}}$ — tensile yield strength; $s^{\text{т}}$ — the boundary on the deformation diagram that separates its elastic part and the yield area ($0 \le s \le s^{\text{т}}$) from the material hardening site ($s > s^{\text{т}}$); $\sigma_*$, $G_*$, $\alpha$, $\beta$ — material constants determined experimentally under simple (proportional) loading.

Under the specified initial conditions for the components $Э_k$ $(k = 1, 3)$ strain vectors **Э** and the value of the angle $\vartheta_1^0$, the basic equations of the model are reduced to the Cauchy problem for each analytical section of the strain trajectory. The trajectories of the strain vector are given, and the trajectories of the stress vector are obtained by integrating the defining relations (1). For the numerical solution and finding the components $S_k$ $(k = 1, 3)$ stress vectors $\boldsymbol{\sigma}$ and values of the approach angle $\vartheta_1$ the Runge–Kutta method of the fourth order of accuracy from the linear algebra package is used MathWorks MATLAB.

## 4. Comparison of calculation results with experimental data

The experimental study was carried out on an automated test machine SN-EVM, which implements a three-parameter effect on the specimen (axial tension–compression, torsion and internal pressure). The author of the article, together with V. I. Gultyaev under the guidance of V. G. Zubchaninov, conducted a series of macroexperiments [14] in the laboratory of mechanical tests of TvSTU. Test specimens in the form of thin-walled cylindrical shells made of 45 steel had a gage length of $l = 110$ mm, the thickness $h = 1$ mm and the diameter of the median surface $d = 31$ mm. The initial isotropy of the specimen material was confirmed with a sufficient degree of accuracy in experiments under simple loading (under tension, compression and torsion). After processing the experimental diagrams, the following values of the material constants (steel 45) were taken in the approximation (4): $\sigma^{\text{т}} = 285$ МПа, $s^{\text{т}} = 9 \cdot 10^{-3}$, $2G = 1{,}577 \cdot 10^5$ МПа, $\beta = 70$, $\alpha = 900$, $\sigma_* = 78{,}8$ МПа, $2G_* = 1619$ МПа.

Figure 1, in the deviator space of deformations $Э_1 - Э_3$ (kinematic loading) shows a plane trajectory of deformation, which corresponds to the process of deformation under a complex combined loading on the specimen of tension-compression and torsion.

In the first, rectilinear section, a proportional extension along the component $Э_1$ to the value is realized $Э_1^* = 1\% = 0{,}01$. In the second section, when the trajectory is broken by an angle $\vartheta_1^0 = 90°$ there is a trajectory of constant curvature in the form of a complete central circle of radius $R = Э_1^*$. In the third section, the strain trajectory smoothly, without a break, passes into the Archimedes spiral, which is crowding at the origin of coordinates, and corresponds to the process of complex disproportionate unloading [14]. The spiral pitch is 0.0025. The curvature of the trajectory on the section of the circle is $\kappa_1 = 100 = \text{const}$, on the 1st turn of the spiral, it changes within $\kappa_1 = 100 \div 133{,}3$; on the 2nd — $\kappa_1 = 133{,}3 \div 200$; on the 3rd — $\kappa_1 = 200 \div 400$ and on the 4th — $\kappa_1 = 400 \div 5026$. In Figure 1 and the following figures, the numbers *1, 2, 3, 4* indicate the experimental points of the beginning of the corresponding turns of the spiral.

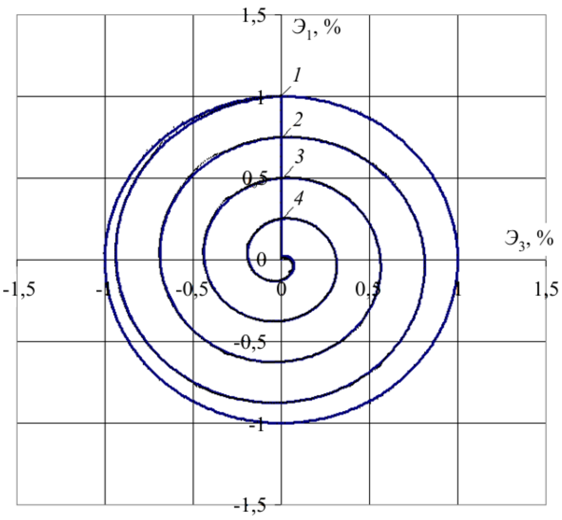


**Fig. 1.** Strain trajectory on the plane $Э_1 - Э_3$

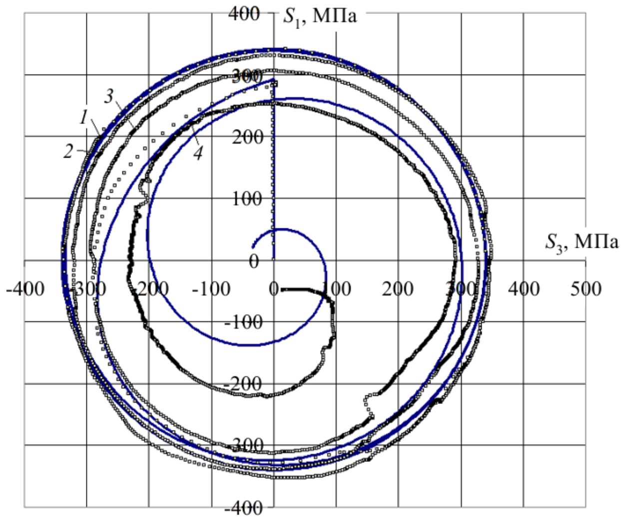


**Fig. 2.** Stress response on the plane $S_1 - S_3$

Figure 2 shows the response to the strain trajectory in the plane $S_1 – S_3$ the deviator space of stresses, Figures 3, 4 show the diagrams $\sigma - s$ and $\vartheta_1 - \Delta s$, describing the scalar and vector properties of the material, respectively. Local strain tension-compression diagrams in components $S_1 - Э_1$ and shear stresses and strains in components $S_3 - Э_3$ they contain Figures 5, 6. The experimental data are indicated by dots, and the calculated data obtained on the basis of the selected approximate model is a solid blue line.

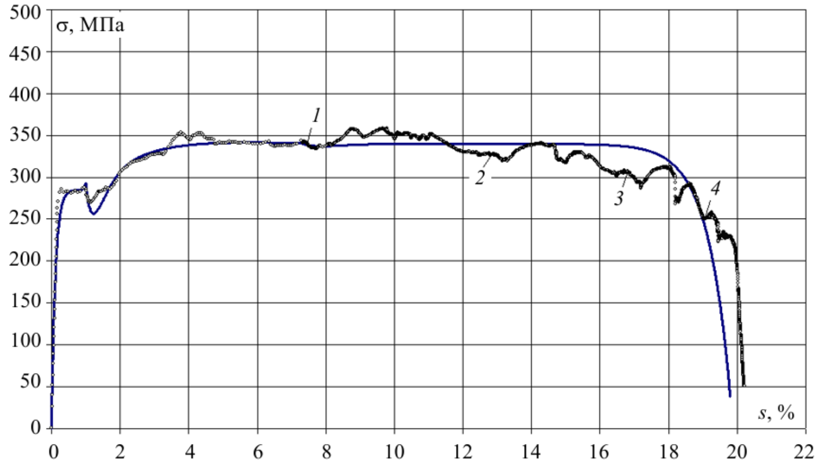


**Fig. 3.** Stress-Strain Diagram $\sigma - s$

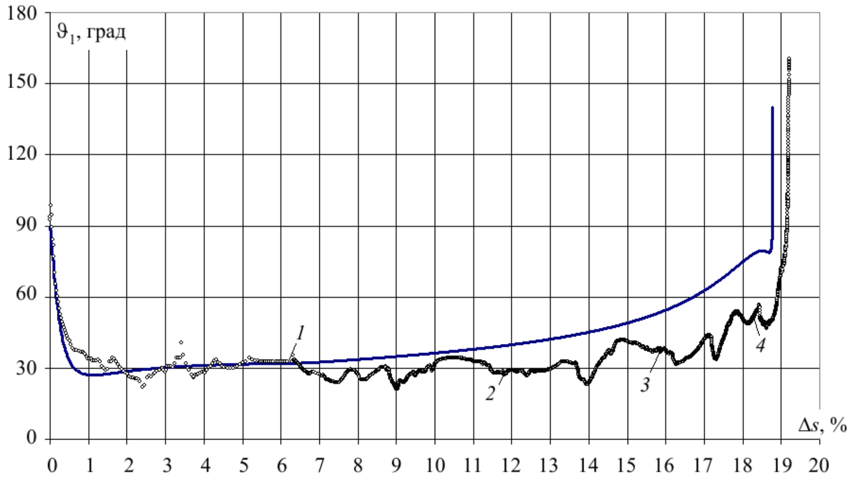


**Fig. 4.** Diagram $\vartheta_1 - \Delta s$ (characteristics of vector material properties)

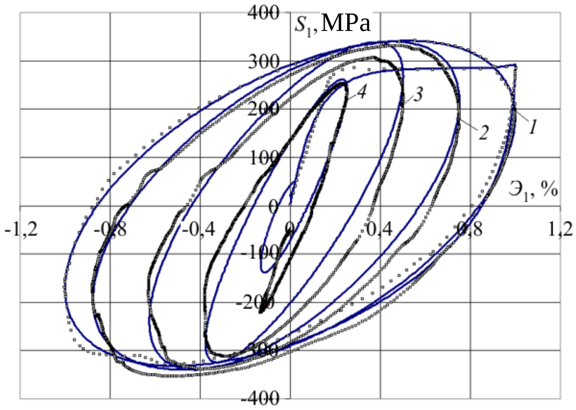


**Fig. 5.** Local Stress-Strain Diagram $S_1 - Э_1$

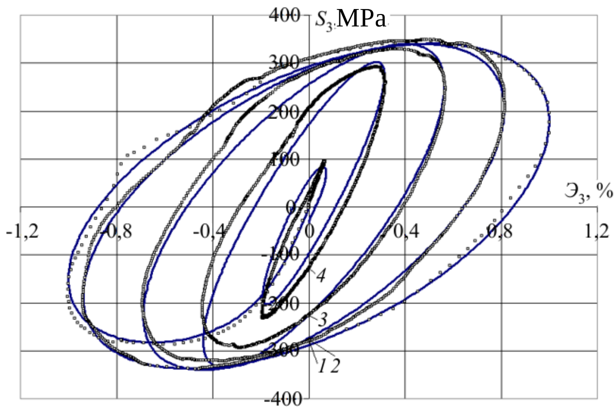


**Fig. 6.** Local Stress-Strain Diagram $S_3 - Э_3$

On the four turns of the Archimedes spiral, the modulus of the stress vector gradually decreases, and here there is a complex disproportionate unloading [3, 14], and starting from the middle of the fourth turn, there is an almost elastic unloading of the material (see Fig. 4), which is qualitatively and satisfactorily quantified by the approximation (2) proposed in the work. The calculated and experimental data on the angle of delay $\vartheta_1$ show that with an increase in the curvature of the trajectory its value increases, and the section of almost elastic unloading of the material corresponds to the value $\vartheta_1 > 90°$. To determine the parameters of approximations $A, b, \gamma, p, q$ in formulas (2), (3), the method described in [16] was used. Parameters $B_1, \alpha_1$ were found by selection based on the condition of compliance with experimental data. The calculation is taken: $A = 370,2$ MPa, $b = 0,125$, $\gamma = 386,8$, $B_1 = 2,9$ MPa, $\alpha_1 = 150$, $p = 1,8$, $q = 0,3$, $s_{\max} = 0,1978$, $\kappa_0 = 100$.

The proposed mathematical model of the theory of processes for curved strain trajectories is approximate since in the expression (2) the functional $\sigma(s)$ it does not depend on the current curvature of the trajectory $\kappa_1$, and from the initial value of the curvature $\kappa_0 = 100 = \text{const}$. As can be seen from the figures, the model shows a good agreement between the calculated and experimental data only for small and medium curvatures of the strain trajectory (see Fig. 2).

At the end of the last turn of the Archimedes spiral, the curvature reaches very large values: $\kappa_1 = 400 \div 5026$, therefore, here we can only speak about the qualitative correspondence of the calculated and experimental data. It is obvious that in order to obtain more accurate calculation results in approximations of plasticity functionals, it is necessary to take into account all the parameters of complex loading. For plane trajectories, this is the length of the trajectory arc $s$, the angles of its fracture $\vartheta_1^0(s)$ and curvature $\kappa_1(s)$.

## 5. Conclusion

The verification of the approximate mathematical model of elastic-plastic processes by comparing the results of numerical calculations with experimental data indicates the correctness of the choice of a model of complex elastic-plastic deformation of steel for this class of curved strain trajectories with analytical sections, such as the Archimedes spiral. There is a qualitative and quantitative agreement between the experimental data and the numerical calculations based on the mathematical model for the trajectories of small and medium curvature.

To refine the results of numerical calculations based on the approximate model, it is necessary to take into account in its functionals all the main parameters of complex loading (parameters of the internal geometry of the strain trajectory). For plane strain trajectories, this is the length of the arc of the trajectory $s$, its curvature $\kappa_1(s)$, and the angles of fracture $\vartheta_1^0(s)$. Failure to take into account the curvature $\kappa_1$ in the functionals approximations may lead to a discrepancy between the calculated results and experimental data for the trajectories of large curvature.

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